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RECLAMATION OF PHOSPHATIC CLAY WASTE PONDS BY CAPPING

Volume 4: Piecewise Linear Computer Modeling of Large Strain Consolidation

Prepared by University of Florida, Department of Civil Engineering,

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Reclamation of Phosphatic Clay Waste Ponds by 'Capping'

Volume 4: Piecewise Linear Computer Modeling of Large Strain Consolidation

Research Project: FIPR 82-02-030

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This report represents the Master of Engineering thesis of Mr. Pedro I. Zuloaga.

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Abstract

Fine-grained mining wastes represent a major disposal problem in the United States. In Florida, as a by-product of the phosphate beneficiation process, over 50 million tons (dry weight) of highly plastic waste clay slurry are produced each year and stored in waste disposal ponds. These waste clays have very poor settling characteristics, with required times of 10 to 20 years for any significant degree of consolidation depending upon pond depths and material parameters. For efficient pond management, containment area sizement and reclamation, the rate and magnitude of consolidation is of paramount importance. Since field monitoring is not practical due to the lengthy time span, centrifugal and computer modelling offer viable alternatives to evaluate these consolidation rates and magnitudes.

Several consolidation computer programs based on the Gibson, England, and Hussey (GEH) theory (1969), and a piecewise linear program based on a spatial representation of finite strain, have been developed. However, GEH programs cannot model non-homogeneous profiles and the piecewise linear program has difficulty modelling initial filling conditions. Furthermore, no multiple layer large strain consolidation model, either finite strain or piecewise linear, has been developed. These drawbacks limit the applicability of computer modelling. Since piecewise linear theory is simpler than GEH theory, and allows for non-homogeneous profiles, a large strain piecewise linear program was developed which allows for any filling scheme in single layer consolidation (UF-McGS) and a method of solution for piecewise linear multiple layer consolidation model is outlined.

Results indicate that the UF-McGS model has excellent agreement with GEH theory for quiescent consolidation, quiescent consolidation with surcharge, and continuous fill. Also, the UF-McGS model agreed with a closed form solution developed for homogeneous quiescent clays.

CHAPTER ONE INTRODUCTION

General

Phosphate is the primary source of phosphorus in inorganic fertilizers with approximately 80% of the United States' requirements and 30% of the world's needs mined in the state of Florida. The matrix of the excavated material is typically composed of l/3 phosphate, l/3 granular materials (sand), and l/3 clays (montmorillonite, attapulgite, illite, and kaolinite) (Bloomquist, 1982). The beneficiation process converts the matrix to a dilute solution from which the phosphate is skimmed, the granular material screened, leaving a dilute clay slurry for disposal. For economical, as well as mining (water recovery) reasons the slurry,which ranges anywhere from 2 - 6% solids content (solids content = W_S/W), is pumped into large retention ponds and allowed to settle/consolidate. However, since the volume of waste slurry generated from the ore extraction process exceeds the volume originally occupied by the matrix, large above ground earth dikes (anywhere from 3 - 15 m high) are needed to impound the clays, as shown in Figure 1.

The adverse impact of this waste disposal technique is: (1) It ties up tremendous quantities of water; and (2) it prevents the development of valuable land (close to 100,000 acres) for agricultural, residential, and/or commercial

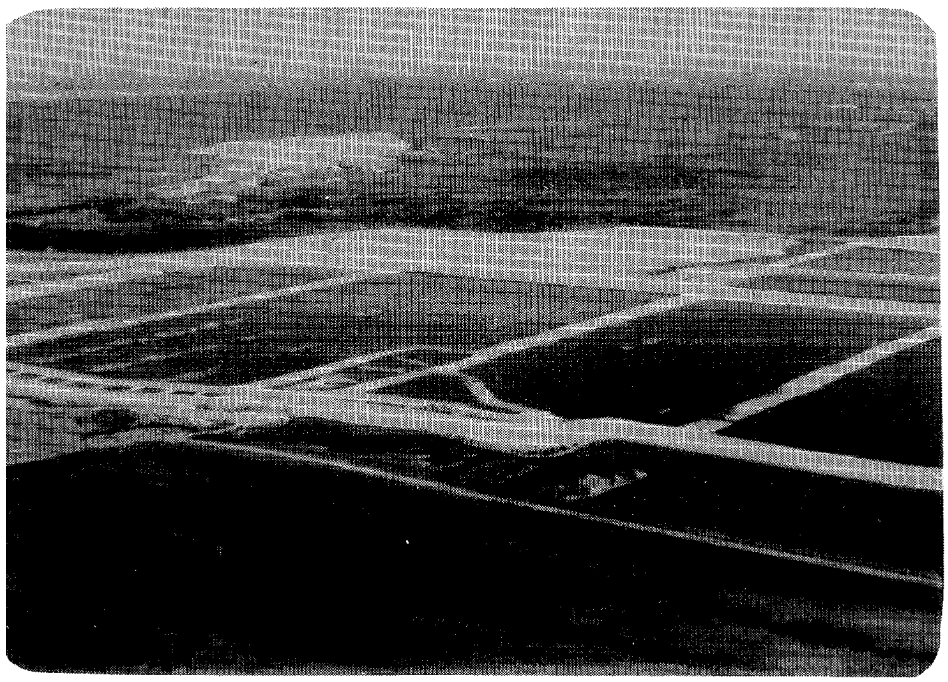

Figure 1.1. Typical Phosphate Impoundment Site

purposes for many years. As a result, significant effort has been expended in finding the most accurate way to predict the rate of consolidation and the final density (final height) of the waste deposits. Such predictions are necessary to estimate the ultimate storage capacity of a disposal area and the time required to achieve its reclamation.

The physical problem associated with large strain settlement may be subdivided into three phases: 1) settling/sedimentation of the suspended fines, 2) hindered settling, and 3) self-weight consolidation of the sediment layer. After placement of a dilute slurry into an impoundment, initial sedimentation occurs until particles begin to interfere with one another (hindered settling), and finally achieve a density where interparticle stress is transferred (consolidation).

Since sedimentation rates for phosphatic clays are several feet per day (Bromwell, 1984), the initial sedimentation appears to occur very quickly compared with the rates of consolidation. In practice, the rate of sedimentation does not strongly influence the design capacity of a waste impoundment, since its useful life, compared with sedimentation time, is usually a matter of years than with a few days to several weeks after deposition. (Bromwell, 1984: Wissa et al., 1983). McRoberts and Nixon (1976) present a theory of sedimentation used to predict the behavior of this phase. When the sedimentation process comes to an end as particles cease to behave as isolated particles or flocs in a suspension, thus acting as a continuum with properties described by traditional parameters, the consolidation phase begins. However, there is a transition period where sedimentation and consolidation occur simultaneously throughout the cross-section of an impoundment. However, little work has been done towards linking sedimentation and consolidation in a single framework (Schiffman et al., 1985).

The driving process behind the consolidation phase for fine grained slurries is governed by the body forces of self-weight (Been and Sills, 1981), as the soil compresses under its own buoyant weight. Typically, phosphatic clay consolidation begins around a solids content of 10% (Bromwell Engineering, 1979), although this value may vary depending upon the initial solids content and the height of the slurry (Keshian et al., 1977). This consolidation phase is critical in impoundment design, and has received considerable research attention in the last decade.

Unfortunately, the rate of settling of waste clays has hampered this research, and field tests are limited by the high cost and the length of time required, for each experiment. Thus, centrifuge and computer modelling offer the only practical techniques for predicting large strain consolidation. However, the former is limited by scaling relationships and the latter by laboratory determination of input parameters.

Due to the large volume change associated with slurried mineral waste consolidation (i.e phosphatic clays), the classic Terzaghi theory, which assumes infinitesimal strain linear consolidation, does not apply. Instead, it is essential that a finite strain nonlinear consolidation theory be used. The most general and accepted programs in phosphate consolidation were developed by Dr. Frank Somogyi who used finite strain, nonlinear consolidation theory in terms of reduced coordinates (to be discussed in Chapter 3) to develop a series of computer programs that can simulate virtually any sequence of filling and quiescent settling, with or without surcharge. These programs have been used extensively for the design and management of disposal areas of several hundred hectares (Carrier et a1.,1983). However, the following characteristics are present and may be of consideration:

1. The constitutive relationships for the material properties (void ratio, effective stress, and permeability) can only be input as power curves.
2. The deposits analyzed must have a homogeneous void ratio profile.
3. The specific gravity of all the solids in the impoundment is assumed to be the same.
4. Only one set of material properties can be analyzed (does not allow for multiple layer analysis)
5. The programs do not possess graphics capabilities.

6. An implicit finite difference scheme is used to solve the governing recurrence formula.
7. In order to model intermittent filling, several programs need to be run.

Noting the drawbacks inherent in representing finite strain theory in reduced coordinates, Yong, et al. (1983), developed a large strain consolidation program utilizing spatial piecewise linear (physical and consolidation parameters are assumed to be constant at a specific time, but are updated as time progresses) theory, which allowed for analysis of non homogeneous profiles. However, the following characteristics are present in this program and may be of consideration:

1. Initial filling conditions are difficult to model.
2. When continuously filling, new material added as time progresses must have the same void ratio as the previous material.
3. Void ratio is defined in terms of clays, bitumen, organics, and non-clays, since the program was originally developed to analyze the consolidation of tailings discharge from tar sands processing (Yong et al., 1983).
4. Only one set of material properties can be analyzed (does not allow for multiple layer analysis).
5. No graphics capabilities, and cumbersome input of the material properties.

6. Stability and convergence restrictions associated with utilizing an explicit finite difference scheme to solve the piecewise linear reccurence formula require lengthy analysis.

Objectives

The following objectives were pursued in this reseach:

1. To modify an existing piecewise linear computer program (Yong) to allow for initial filling and to improve its existing features and applicability.
2. To compare spatial with reduced representation finite strain nonlinear consolidation theory whenever applicable.
3. To predict a series of model ponds.
4. To develop a multiple layer piecewise linear large strain consolidation model.

Scope

Based upon Yong's piecewise linear computer code as documented by Hernandez (1985), a general one dimensional large strain consolidation program was developed, "UF-McGS" (University of Florida - McGill University Single Layer), with the following features:

1. Interactive and batch input.
2. Input and output parameters in three unit systems.
2. Built-in batch editor.

3. Lotus 1-2-3 compatability for graphics.
4. General definition of void ratio, based on clay, sand, and other solids.
5. Calculation of the average degree of consolidation based on void ratios.
6. Print control parameters selected by the user.
7. Automatic generation of power curves to determine the relationship between the void ratio, effective stress,and permeability, or allow for manual input of relationships other than exponential.
8. Allowance for analysis of any sequence of filling and quiescent settling, including surcharge loading.
9. Two output files, one describing the effective stress, pore pressure, and void ratio distribution at particular times determined by the user and the other showing the change in height, average degree of consolidation, and average solids content with time.

This model was verified against an available closed form solution, and the Somogyi model, when applicable. These verifications were made by predicting a series of eight ponds. Also, parametric studies were performed so as to determine the most efficient way of running the UF-McGS and Somogyi models. Interactive input capabilities, including a built-in batch file editor for Somogyi's

QSUS program, and expansion of the dimension statements for all Somogyi programs were added so as to make parameteric studies easier. A piecewise linear multiple layer consolidation model is developed, and a method of solution is outlined.

CHAPTER TWO
LITERATURE REVIEW

Introduction

This chapter will discuss the evolution of one-dimensional consolidation theories, beginning with "classical" theory, as developed by Terzaghi; through finite strain theory in terms of reduced coordinates, as developed by Gibson, England, and Hussey (GEH); and piecewise linear theory in terms of convective coordinates, as developed by Olson and Ladd, and later used by Yong.

Conventional Theory of Consolidation

The theory of consolidation is a continuum theory designed to predict the progress of deformation of an element of a porous material when this element is subjected to an imposed disturbance. Its origins can be traced back to the one-dimensional theory of consolidation formulated by Karl Terzaghi in 1923. This formulation was a finite strain theory, but it was assumed that the compressibility and the reduced coefficient of permeability were constant. The latter is defined as $k/(1+e)$, where k is the conventionally measured coefficient of permeability, and e is the current void ratio (Schiffman et al., 1985).

The original one-dimensional theory of consolidation was reformulated in 1942 by Terzaghi into what is known today as "conventional" or "classical" theory which assumes infinitesimal strains, constant compressibility, and permeability. However, it is widely recognized that conventional theory assumptions are only approximately satisfied, and the error arising from such assumptions will depend on the magnitude of the load increment, and of the void ratio changes (Gibson et al., 1967). Studies by Gibson et al. (1981), Schiffman et al. (1985), and McVay et al. (1986), show that for highly compressible saturated soils, such as the ones encountered in phosphatic waste disposal areas, conventional theory will seriously overestimate the time of consolidation and underestimate settlement and the amount of excess pore pressure at a given time, due to the assumed rigidity of the skeleton.

Recognizing the limitations of classical theory, as discussed by Gibson et al. (1967), several authors such as Richart (1957), Lo (1960), Schiffman and Gibson (1964), Davis and Raymond (1965), Janbu (1965), and Barden and Berry (1965), attempted to extend the classical theory to account for the variation of permeability and compressibility based on small strain theory. For example, Schiffman and Gibson treated the nonlinearity of stress-strain and stress-permeability as a spacially dependent problem (Hernandez, 1985). Davis and Raymond extended the classical theory to a nonlinear one by assuming the coefficient of consolidation to be constant while the compressibility and permeability are both allowed to decrease with increasing pressure. However, both these attempts were unsuccessful in developing a general one-dimensional consolidation equation since the variation of permeability and compressibility are likely to be of real importance only if the void ratio changes and strains are appreciable.

Finite Strain Theory

Generalizations of consolidation theory aimed at eliminating the restriction of small strains, while taking into account changes in soil compressibility and permeability, were independently established in the one-dimensional nonlinear finite strain theory by Mikasa in 1963, and Gibson, England, and Hussey (GEH) in 1967. Pane and Schiffman (1981) show that the GEH theory and the Mikasa theory differ in their underlying assumptions in only one respect. Mikasa's theory is limited to the case where the deposit consolidates under its own weight with or without an imposed surface loading after rapid sedimentation, whereas GEH theory is unrestricted as to its initial condition. The general form of the governing relationship as presented by Gibson et al. (1967) is:

$$[\frac{\gamma_s}{\gamma_w} - 1]\frac{d}{de}[\frac{k(e)}{1+e}]\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}[\frac{k(e)}{\gamma_w(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}] + \frac{\partial e}{\partial t} = 0 \qquad (2.1)$$

where

γ_s = the unit weight of solids

γ_w = the unit weight of water

k = permeability

e = void ratio

σ' = effective stress

$\frac{\partial}{\partial z}$ = spatial derivative in terms of reduced coordinates (reviewed in Chapter 3)

It should be noted that the $\partial e/\partial t$ term shown in the above equation should be expressed as a material derivative (McVay et al., 1986).

Based on equation 2.1, a number of computer programs have been developed for the purpose of solving one-dimensional nonlinear finite strain consolidation problems based on reduced coordinates. For example, a series of programs have been developed at the University of Colorado and at the Waterways Experiment Station which assume that the void ratio-effective stress and void ratio-permeability relationships are exponential functions, and the highly non-linear term

$$g(e) = -\frac{k(e)}{\gamma_w(1+e)}\frac{d\sigma'}{de} \tag{2.2}$$

is a constant. Furthermore, these programs are based on a governing relationship where the void ratio is the dependent variable, and utilizes an explicit finite difference scheme (Gibson et al., 1981). The programs developed at the University of Colorado were developed primarily for research purposes (Gibson et al., 1981), while the second generation programs developed at the Waterways Experiment Station were designed primarily to assist in the planning of dredge fill operations (Cargill, 1983). It should be noted, that the finite difference method consists of substituting finite ranges for differentials in the governing equation, and will be discussed further in Chapter 4. Recently, the University of Colorado has developed a series of similar programs utilizing the method of lines (Pane, 1985) which lifts the restriction with respect to the form of the void ratio-effective stress and void ratio-permeability relationships (Caldwell et al., 1984; Schiffman et al., 1984). Finally, the Somogyi model, which is used in this thesis to validate the piecewise linear models, is based on Koppula's (1970) manipulation of equation 2.1,

$$\frac{\partial}{\partial z}\left[-\frac{k}{\gamma_w(1+e)}\frac{\partial u}{\partial z}\right]+\frac{de}{d\sigma'}\frac{\partial \sigma'}{\partial t}=0 \tag{2.3}$$

Spatial Piecewise Linear Theory

The most noticeable drawback observed in finite difference computer programs utilizing GEH theory is their inability to model profiles with non-homogeneous void ratios. Piecewise linear theory in terms of convective coordinates, introduced by Olson and Ladd in 1979 and later used by Yong et al. (1983), avoids this simplification by modelling finite strain using an updated Lagrangian finite difference scheme iterative solution which refers all static and kinematic variables to the configuration at time t. This requires not only that the material parameters (permeability, effective stress, void ratio) but representative geometry be updated after each time step as well (McVay et al., 1986). Piecewise linear modelling allows one to consider continuous loading, non-linear material properties, and non-homogeneous soils. To minimize errors, a double precision technique should be used. The Yong model, which was developed by Yong et al. (1983), is the basis for the piecewise linear models (UF-McGS and UF-McGM) developed in this thesis. This model was chosen because of its potential for expandability, and flexibility in modelling any non-homogeneous condition.

CHAPTER THREE
MATHEMATICAL FORMULATION

As discussed in the literature review, several authors have developed mathematical formulations to model large strain consolidation; however, until McVay et al. (1986), no single work acknowledged that all these formulations stemmed from a single general theory of consolidation. The following mathematical derivation of the theory of consolidation is based on this work: it will show that the governing equations for the finite strain program (Somogyi) in terms of reduced coordinates and the spatial piecewise linear programs (UF-McGS, Yong) can be derived from the same general equation.

Coordinate Systems

The use of a fixed reference system to describe large strain consolidation is impractical due to the relative large movement of the top boundary of the consolidating layer. Therefore, to simplify the required mathematics, a coordinate system which moves with the layer is needed (Cargill, 1982). Reduced coordinates (z), introduced by Terzaghi (1923) and later used by Ortenbland (1930) and McNabb (1960), as indicated by Schiffman et al. (1985) and McVay et al. (1986),describe the consolidating layer at any time only in terms of the volume of solids, and thus are suited to describe large strain consolidation because they are independent of time and the amount of strain.

Although reduced coordinates will be used to derive the Somogyi model, the large strain consolidation problem can be best described physically by combining the Lagrangian and convective coordinate systems. In the Lagrangian approach, the position vector (a) is attached to the particle, so the location of the particle can be described at any time during consolidation. During consolidation ($t>0$), the location of a soil particle will depend on both the initial position and the time elapsed. Thus, convective coordinates (ξ), which fix attention on a given region of space instead of on a given body of matter, and which are dependent on the Lagrangian coordinate and time, are applicable. Figure 3.1 describes a soil particle within a consolidating layer in terms of Lagrangian and convective coordinates; and Figure 3.2, shows the difference between the Lagrangian, convective, and reduced representations.

The three coordinate systems described herein can be related to each other using the following conversions:

$$\frac{\partial a}{\partial \xi} = \frac{1 + e_o}{1 + e} \tag{3.1}$$

$$\frac{\partial z}{\partial a} = \frac{1}{1 + e_o} \tag{3.2}$$

$$\frac{\partial \xi}{\partial z} = 1 + e \tag{3.3}$$

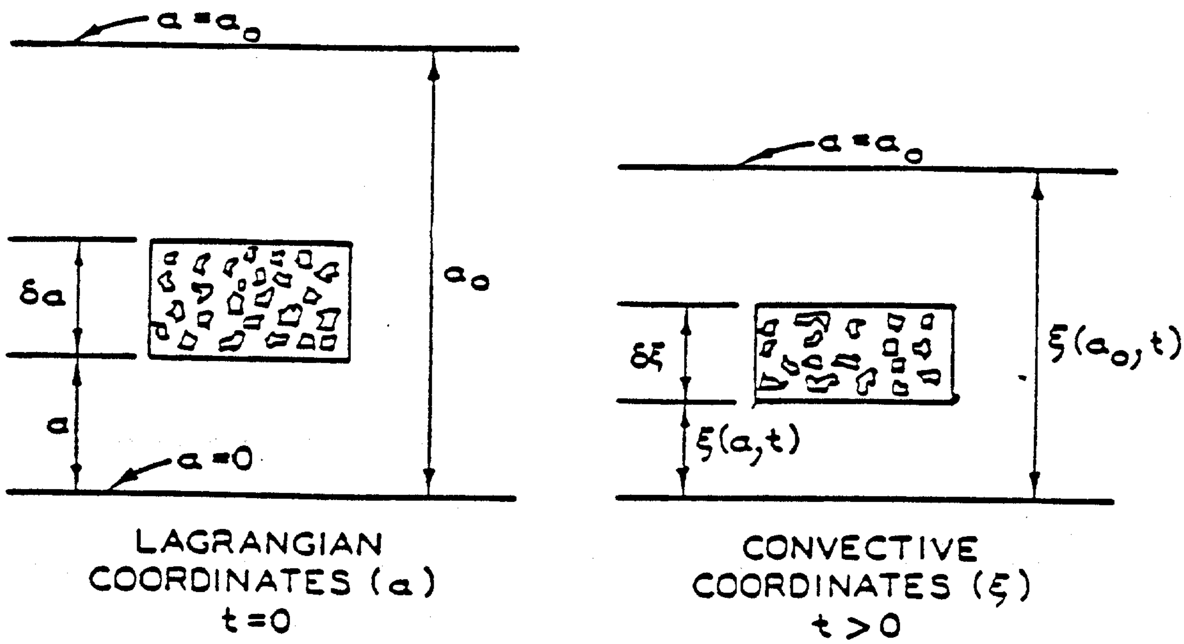

Figure 3.1. Geometrical Description of Soil Elements in Lagrangian and Convective Coordinates (Cargill, 1982).

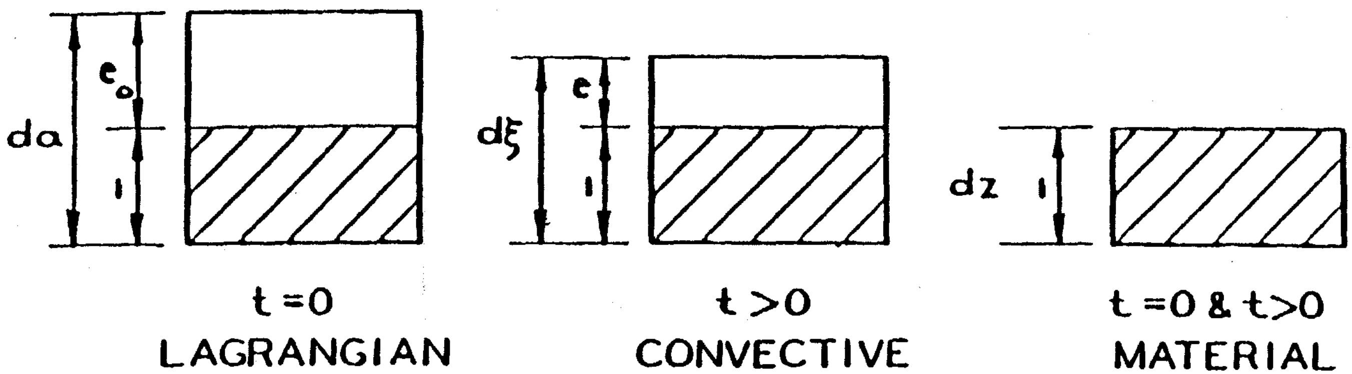

Figure 3.2. Comparison of Differential Soil Elements in Lagrangian, Convective, and Reduced Coordinates (Cargill, 1982).

Theorv of Consolidation

McVay et al. (1986) recognized that the mathematical description of one dimensional consolidation may be most appropriately developed from the continuum theory of mixtures as first put forth by Truesdell and Toupin (1960), and later used by Bear (1979). The cornerstones of this development are:1) conservation of mass and momentum of the solid and fluid phases and 2) a constitutive relationship. Specifically, mass conservation of the fluid, assuming the fluid incompressible, may be expressed as

$$\frac{\partial q}{\partial \xi} + \frac{\partial n}{\partial t} = 0 \qquad (3.4)$$

where

q (exit fluid velocity) = $V \times n$ (3.5)

V = seepage velocity

n = the porosity

while the volume conservation of solids, assuming the solid grains incompressible, may be expressed as

$$\frac{\partial}{\partial t}[1 - n] + \frac{\partial}{\partial \xi}[(1 - n)V_s] = 0 \qquad (3.6)$$

where

V_s = the velocity of solids

and the balance of momentum of the fluid phase, assuming isotropy, linearity, and neglecting inertia, may be expressed as

$$V_f = n(V - V_s) = -\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi} \tag{3.7}$$

where

u = excess pore pressure at ξ, t

V_f = fluid velocity relative to the the solids

Equation 3.7 was discussed by Raats and Klute (1965), who noted that this equation may be regarded as Darcy's law appropriate to flow of a fluid phase completely filling the pores of a solid phase which might move in a nonrigid manner.

By combining equations 3.4 through 3.7, as suggested by Bear (1979), this equation can be obtained;

$$\frac{\partial}{\partial \xi}\left[\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi}\right] = \frac{1}{(1-n)}\left.\frac{Dn}{Dt}\right|_X \tag{3.8}$$

As McVay et al. (1986) points out, it is worth noting that $\partial u/\partial \xi$ represents a spatial derivative and characterizes the local rate of change of excess pore pressure with respect to ξ, whereas Dn/Dt represents a material derivative and shows the change in porosity as one follows the particle.

Defining the porosity in terms of void ratio

$$n = \frac{e}{1 + e} \tag{3.9}$$

its material derivative may be represented as

$$\left.\frac{Dn}{Dt}\right|_X = \frac{1}{(1+e)^2}\left.\frac{De}{Dt}\right|_X \tag{3.10}$$

where

$$\left.\frac{De}{Dt}\right|_X = \frac{\partial e}{\partial t} + V_s\frac{\partial e}{\partial \xi}$$

$\frac{\partial e}{\partial t}$ = spatial time derivative of the void ratio

V_s = velocity of solids

$\frac{\partial e}{\partial \xi}$ = spatial derivative of the void ratio

Substituting equation 3.10 into equation 3.8, the following one-dimensional consolidation equation is obtained

$$\frac{\partial}{\partial \xi}\left[-\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi}\right] + \frac{1}{(1+e)}\left.\frac{De}{Dt}\right|_X = 0 \tag{3.11}$$

In order to relate stress changes to corresponding strains, a constitutive relationship is necessary. Terzaghi (McVay, 1986a) assumed the void ratio to be expressed explicitly in terms of the effective stress alone

$$e = f\ (\sigma') \tag{3.12}$$

Taylor and Merchant (1940) and Gibson and Lo (1961) objected to Terzaghi's constitutive relationship, and proposed that the void ratio be expressed in terms of effective stress and time by stating that part of the decrease in void ratio is due to secondary compression, or volume decrease at a constant effective stress.

However, to the author's knowledge, the norm when describing one-dimensional consolidation is to assume, as Terzaghi did, the void ratio to be expressed explicitly in terms of void ratio when modelling large strain consolidation (Been and Sills, 1981; Gibson et al., 1967: Gibson et al., 1981; Monte and Krizek, 1976; Yong and Ludwig, 1984). The Somogyi program (finite strain in reduced coordinates), and the UF-McG models (piecewise linear), both assume that Terzaghi's assumption, equation 3.12, is valid.

By taking the material derivative of equation 3.12,

$$\left.\frac{De}{Dt}\right|_X = \left.\frac{de}{d\sigma'}\frac{D\sigma'}{Dt}\right|_X \qquad (3.13)$$

and substituting it into equation 3.11, an equation describing the consolidation process in terms of effective stresses is obtained

$$\frac{\partial}{\partial \xi}\left[\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi}\right] = \left.\frac{1}{(1+e)}\frac{de}{d\sigma'}\frac{D\sigma'}{Dt}\right|_x \qquad (3.14)$$

If the soil deposit remains submerged, as is assumed in large strain slurry consolidation, hydrostatic pressure contributes an equal amount to both total stress and pore fluid pressure; therefore, effective stress can be written as

$$\sigma' = \sigma_b - u \qquad (3.15)$$

where

$\sigma_b = \sigma - u_o$

σ_b = buoyant stress

σ = total stress

u = excess pore pressure

u_o = hydrostatic pore pressure

The material derivative of equation 3.15 may be expressed as

$$\left.\frac{D\sigma'}{Dt}\right|_X = \left.\frac{D\sigma_b}{Dt}\right|_X - \left.\frac{Du}{Dt}\right|_X \tag{3.16}$$

Substituting equation 3.16 into equation 3.14 results in a general one-dimensional consolidation equation in terms of buoyant stresses and excess pore pressures

$$\frac{\partial}{\partial \xi}\left[\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi}\right] - \frac{1}{(1+e)}\frac{de}{d\sigma'}\left.\left[\frac{D\sigma_b}{Dt} - \frac{Du}{Dt}\right]\right|_X = 0 \tag{3.17}$$

Governing Equations

The governing equations for conventional theory, the Somogyi model, and piecewise linear models may be derived by manipulating equation 3.17.

Conventional Theory

Terzaghi's conventional one-dimensional consolidation theory

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial \xi^2} \tag{3.18}$$

where

c_v (coefficient of consolidation) = $\frac{k\ (1+e)}{a_v\ \gamma_w}$

a_v (coefficient of compressibility) = $-de/d\sigma'$

is readily obtainable by applying the following restrictions to equation 3.17:

1) Constant permeability

2) Quiescent consolidation ($\frac{D\sigma_b}{Dt} = 0$)

and 3) Rigid soil skeleton ($V_s \frac{\partial u}{\partial \xi} = 0$)

$$\frac{k(1+e)}{\gamma_w a_v} \frac{\partial^2 u}{\partial \xi^2} = \left.\frac{Du}{Dt}\right|_X$$

$$c_v \frac{\partial^2 u}{\partial \xi^2} = \left.\frac{Du}{Dt}\right|_X$$

$$c_v \frac{\partial^2 u}{\partial \xi^2} = \frac{\partial u}{\partial t} + V_s \frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial t}$$

Somogyi Model

Somogyi (1980) formulated the governing equation for his finite strain computer program by considering equation 3.17 in terms of reduced coordinates. Koppula (1970) also arrived at a reduced representation of equation 3.17 without expanding the terms described in equation 3.16 by manipulating the equations of equilibrium, continuity, and flow, as originally derived by Gibson et al. (1967).

Substituting equation 3.3 into equation 3.14, the following equation may be obtained

$$\frac{\partial}{\partial z}\left[\frac{k}{\gamma_w}\frac{\partial u}{\partial z}\right] = \frac{1}{(1+e)}\frac{de}{d\sigma'}\frac{D\sigma}{Dt}\bigg|_X \tag{3.19}$$

In terms of reduced coordinates, the buoyant stress at any point depends only the amount of solid particles above that point. If additional soil particles are not deposited (quiescent consolidation), the buoyant stress at any reduced coordinate depth is time independent, and may be represented as

$$\sigma_b(z) = \gamma_b(z_{top} - z) \tag{3.20}$$

where

$\sigma_b = (G_s - 1)\gamma_w$

G_s = specific gravity of solids

In the case of either initial or continued deposition, the buoyant stress can be written as

$$\sigma_b = \sigma_b + \Delta\sigma_{b_i} \tag{3.21}$$

where

σ_{b_i} = initial variation in buoyant stress

Combining equations 3.15, 3.20, and 3.21, material variation of effective stress with time, may be given by,

$$\frac{D\sigma'}{Dt} = (G_s-1)\gamma_w \frac{d\Delta Z}{dt} - \frac{Du}{Dt} \tag{3.22}$$

Substituting the above equation into equation 3.19 yields

$$\frac{\partial}{\partial z}\left[- \frac{k}{\gamma_w} \frac{\partial u}{\partial z}\right] + \frac{de}{d\sigma'} \left[(G-1)\gamma_w \frac{d(\Delta z)}{dt} - \frac{Du}{Dt} \right] = 0 \tag{3.23}$$

This is the general equation used by Somogyi to describe the behavior of excess pore pressure during consolidation. However, in order to solve this equation numerically, two material functions describing compressibility and permeability are necessary. Roma (1976), see Figure 3.3, reported that the best relationship between equilibrium void ratio and effective stress for phosphatic clays can be best described by the following power fit

$$e = A\sigma'^B \tag{3.24}$$

Likewise, Keshian et al. (1977), and more recently, Wissa et al. (1983), (see Figure 3.4) indicated that the following power fit showed a good correlation between permeability and void ratio

$$k = Ce^D \tag{3.25}$$

Substituting equation 3.24 into 3.23, rearranging, and simplifying, Somogyi obtained thefollowing equation

$$\frac{Du}{Dt} + \frac{\sigma'^\beta}{\alpha}\left[\frac{k}{1+e}\right]\frac{\partial^2 u}{\partial z^2} + \frac{\sigma'^\beta}{\alpha}\frac{\partial}{\partial z}\left[\frac{k}{1+e}\right]\frac{\partial u}{\partial z} = \gamma_b\frac{d(\Delta z)}{dt} \tag{3.26}$$

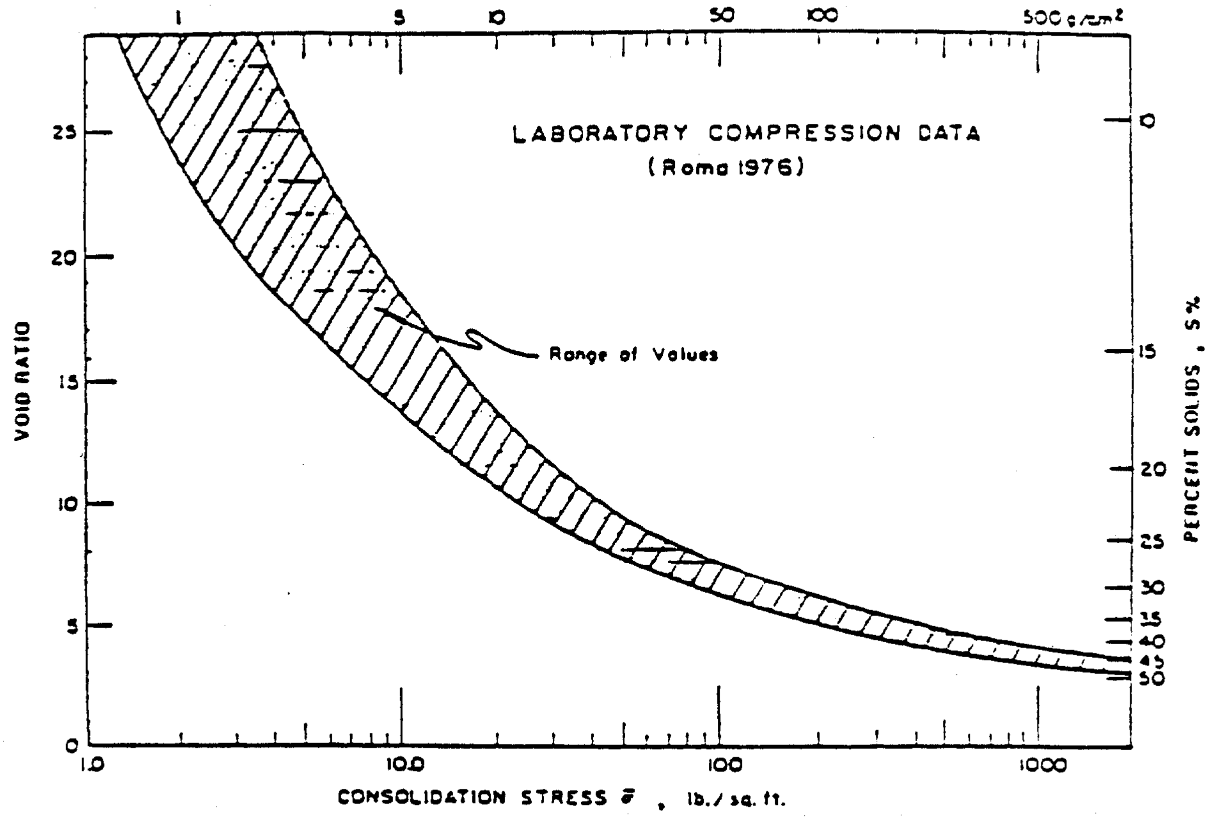

Figure 3.3. Void Ratio vs. Effective Stress for Phosphatic Clays (Bromwell Engineering, 1979).

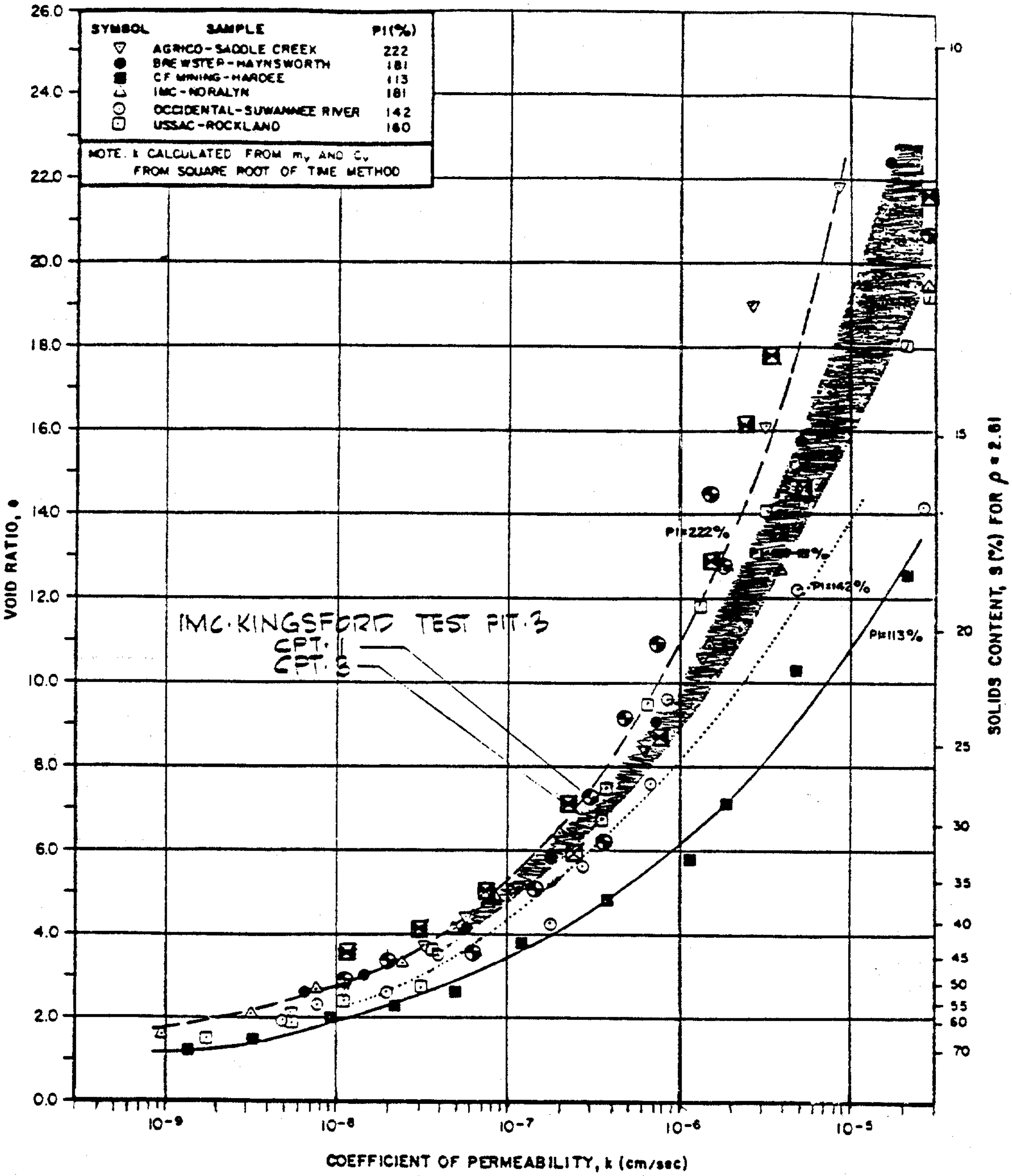

Figure 3.4. Void Ratio vs. Permeability for Phosphatic Clays (Wissa et al., 1983).

where

$\alpha = AB\gamma_w$

$\beta = 1-B$

Equation 3.26, combined with equation 3.25, permits the solution of large strain consolidation using finite difference, which will be discussed in the next chapter.

Piecewise Linear Models

Equation 3.17, which may be derived from equation 3.14 (the latter also being the basis for Somogyi's governing equation), serves as the governing equation for the UF-McG models. The Yong model simplifies this equation by not considering the change in buoyant stress with time term implicitly--see Chapter 4 for explanation, and thus has the following governing equation (Yong and Ludwig, 1984)

$$\frac{Du}{Dt} = \frac{1+e}{\gamma_w}\frac{d\sigma'}{de}\frac{\partial}{\partial\xi}(k(e))\frac{\partial u}{\partial\xi} + \frac{k(e)(1+e)}{\gamma_w}\frac{d\sigma'}{de}\frac{\partial^2 u}{\partial^2\xi} \qquad (3.27)$$

By analyzing equations 3.17 and 3.26, it can be concluded that the only difference in the governing equation between the Somogyi and UF-McG models is the coordinate system chosen for solution. As previously stated, the Somogyi model solves equation 3.24 in terms of reduced coordinates, while the UF-McG models solve equation 3.17 in terms of convective coordinates. In order to account for the buoyant stress being time dependent, as is the case with convective coordinates, the UF-McG models, as well as the Yong model, require that not only the material parameters (void ratio, effective stress, and permeability), but the representative geometry be updated as time progresses.

The UF-McG models combine equation 3.17 with material functions relating void ratio to effective stress and void ratio to permeability, in order to solve large strain consolidation problems using the finite difference technique.

Closed Form Solution

To our knowledge, there exists only one closed form solution which allows for the calculation of phosphatic clay settlement (McVay, 1986b) and due to the non-linearity of the problem, there are no closed form solutions capable of calculating the rate of large strain consolidation.

The closed form solution which will now be discussed was developed for quiescent clays with homogeneous void ratio profiles by McVay (1986b) and expounded on by Townsend (1986b),to allow for sand surcharge.

Assuming that equations 3.24 and 3.25 properly describe the material characteristics of an impoundment, and that one-dimensional consolidation is modelled

$$\frac{\Delta e}{1+e_o} = \frac{\Delta H}{H_o} \tag{3.28}$$

Rearranging and integrating throughout the impoundment profile

$$H = \int_{H_i}^{H_f} \frac{\Delta e}{1+e_o}\, dz \tag{3.29}$$

where

z = height of clay = $H_f - H_i$

If $\Delta e = e_o - e_f$, (3.30)

and

$$e_f = A\sigma'^{B}, \tag{3.31}$$

then

$$\Delta e = e_o - A\sigma'^{B} \tag{3.32}$$

Since $\sigma' = (\gamma_b)z$, then,

$$\Delta e = e_o - A(\gamma_b z)^{B} \tag{3.33}$$

Then, substituting equation 3.33 into equation 3.29 and integrating, the following closed form solution is obtained for quiescent clays.

$$\Delta H = \frac{1}{1+e_0}\left[e_0(H_f - H_i) - A(\gamma_b)^{1+\beta}\{H_f^{(1+\beta)} - H_i^{(1+\beta)}\}\right] \tag{3.34}$$

Where H_i = top of pond depth corresponding to the stress at e_o, and $H_f = H_i$ + depth of clay, z. Hi can be determined by rearranging equation 3.24 as

$$\sigma' = (e_0/A)^{1/\beta} \tag{3.35}$$

and remembering $H_i = \sigma'/\gamma_b$ (3.36)

In the case of a sand cap

$$\sigma' = (\gamma_{bc})\, Z_c + q \tag{3.37}$$

where

z_c = height of clay = $(z_f - z_i)$

γ_{bc} = clay buoyant unit weight

and $q = (\gamma_{bc}) z_s$ = (sand buoyant unit weight)(height of sand cap)

Substituting equation 3.37 into equation 3.31, and the latter into 3.30

$$\Delta e = e_o \, [(\gamma_{bc})Z_c + q]^{\beta} \qquad (3.38)$$

Finally, substituting equation 3.36 into equation 3.29 and integrating between the limits of z_f and z_i the following closed form solution is obtained for quiescent clays with a sand surchage (eqn. 3.39),

$$\Delta H = \frac{1}{1+e_o} \{e_o(Z_f - Z_i) - \frac{A}{(\gamma_{bc})(1+\beta)} [((\gamma_{bc}) Z_f + q)^{1+\beta} - ((\gamma_{bc}Z_i) + q)^{1+\beta}]\} \qquad (3.39)$$

Where z_i = top of pond depth corresponding to the stress e_o similar to H_i and can be found using the approach of equations (3.35) and (3.36), and $z_f = z_i$ + depth of clay, z_c.

CHAPTER FOUR
SOMOGYI AND YONG MODELS

Finite Difference

General

Since very limited closed form solutions are available to predicts the settlement of slurries, numerical approximations must be made. The computer programs described in Chapters 4 and 5 utilize the finite difference technique, which consists of replacing continuous derivatives in the governing equation by the ratio of changes in the variable over small but finite increments, establishing rules to ensure that the method of solution is stable and does not lead to cumulative errors. The governing equation expressed in terms of finite differences is called a recurrence formula. The latter may be solved explicitly (UF-McG) or implicitly (Somogyi), by relating forward, backward, or central differences obtained from the Taylor series expansion. A detailed description of finite difference will not be reviewed, since the general method is covered in a number of books (Desai and Christian, l977;Ames, 1969). However, several features of the finite difference method related to consolidation which are used in this report will be summarized in the next sections.

Since finite difference is an approximate solution to a differential equation, the solution must converge and be stable. Convergence means that the results of a finite difference method must approach the correct values as Δt and Δz both approach zero, while stability means that errors introduced at one stage of the calculations do not cause increasingly large errors as the computations are continued, but eventually damp out.

Explicit and Implicit Formulations

As discussed previously, finite difference problems may be solved explicitly or implicitly. For example, if the updated dependent variable is the pore pressure u_i, the explicit scheme would approximate a solution for u_i at time level "t+1" in terms of the surrounding known values of u_i at the previous time level "t". The explicit procedures are relatively straightforward, permit step-by-step evaluation of u_i directly, and do not require solution of simultaneous equations. However, explicit formulations have three major drawbacks: 1) to ensure stability, only very small increments in the time variable are permissible, 2) the updated dependent variable will only be influenced by the immediately surrounding values from the previous time step, and 3) there is a region within which boundary values have no influence, as shown in Figure 4.1. These drawbacks are solved by implicit finite difference approximations, which require the solution of simultaneous equations at time level "t+1", and include the influence of known boundary values at

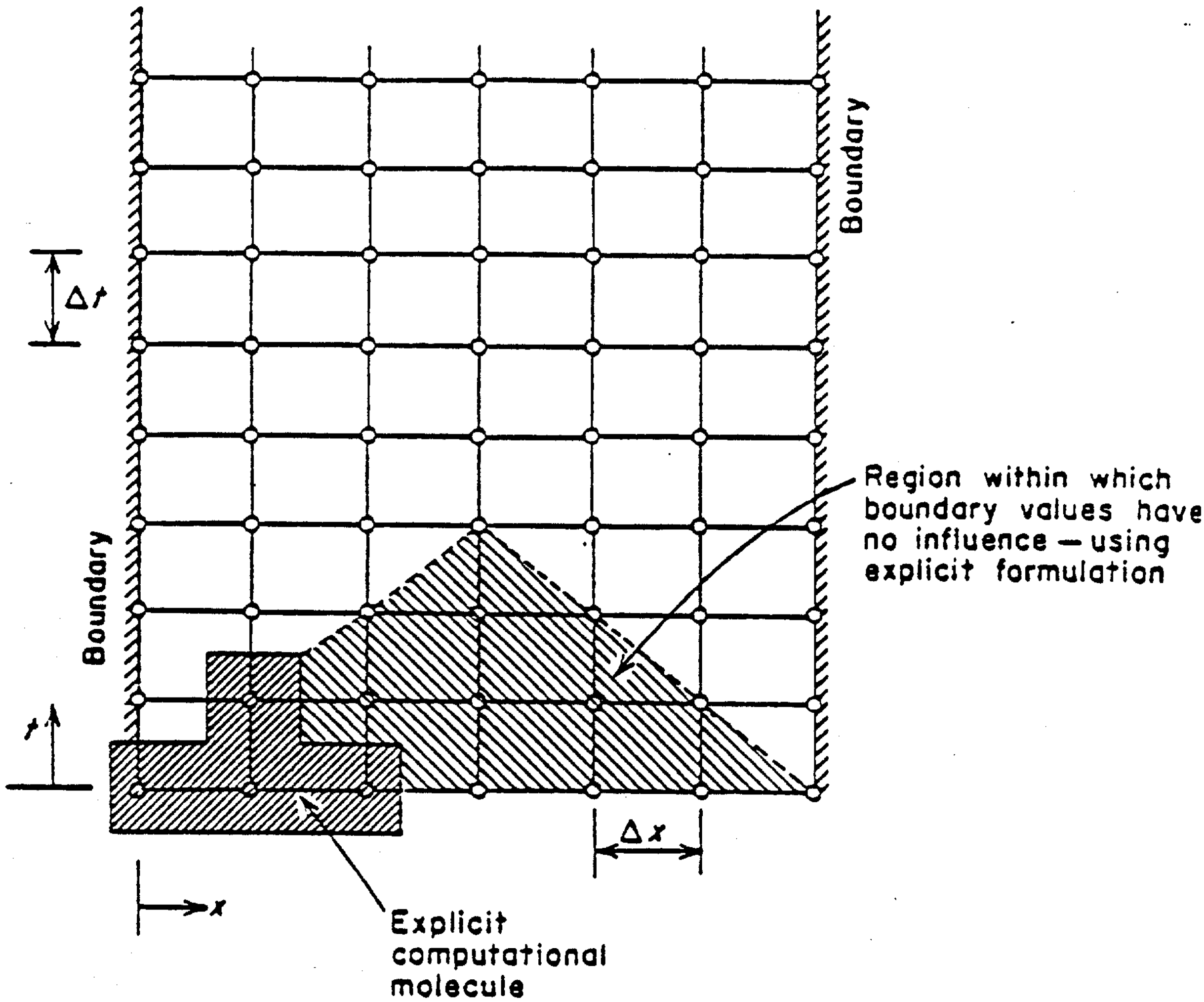

Figure 4.1. Depiction of Major Drawback Inherent with Explicit Finite Difference Schemes (Ketter and Prawel, 1969).

that same time. An implicit recurrence formula is one in which two or more unknown values at time "t+l" are specified in terms of known values at time "t" (and "t-l","t-2",..., if necessary) by a single application of the expression (refer to Figure 4.2). If there are M unknown values at time "t+l", the recurrence formula must be applied M times across the length of the row. The resulting system of M simultaneous equations specifies the M net values implicitly (Ames, 1969, p. 50). However, this method can have drawbacks, which will be discussed in the next section.

Numerical Solution

Somogyi Model

General. Somogyi chose a fully implicit method to solve the excess pore pressures of equation 3.26 because of the scheme's inherent stability, convergence, and ease of programming. Once the excess pore pressures at a new time are solved, the distribution of effective stresses is calculated from equation 3.15, the corresponding void ratios and permeabilities are obtained from equations 3.24 and 3.25, and the solution is advanced. However, here lies the major drawback associated with using a fully implicit technique when modelling large strain consolidation: the values of void ratio and permeabilities at the new time step are required for the solution of the excess pore pressure. These values are unknown. Somogyi's solution to this problem was to employ a time increment sufficiently small so

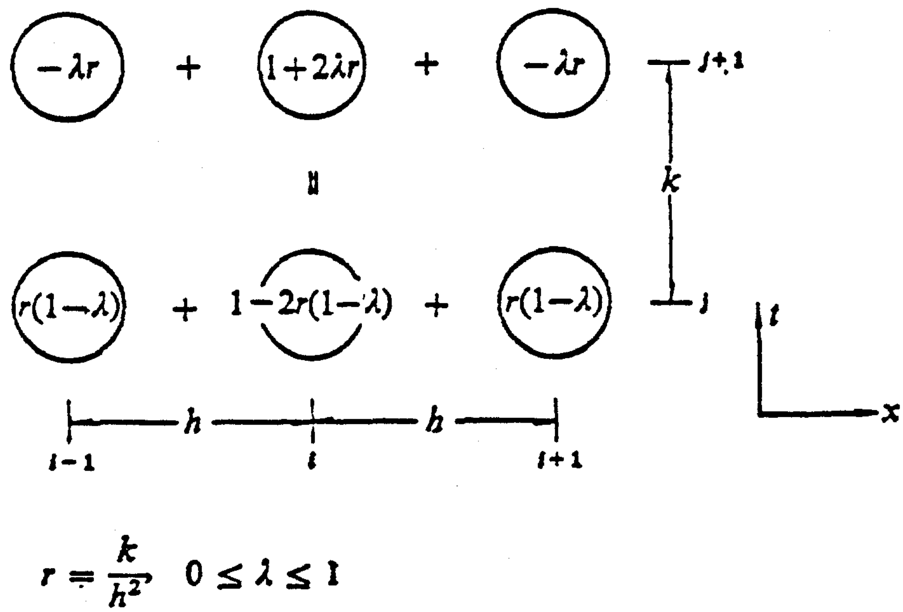

Figure 4.2. Implicit Finite Differences Mesh (Ames, 1969).

that little variation in the above parameters would occur; the unknown values at the new time can then be replaced by known values at the previous time. This in effect causes changes in void ratio and permeability to lag one time-step behind changes in excess pore pressure. Fortunately, the stability of the implicit technique ensures that errors thus introduced will eventually decay. Desired accuracy can be obtained by reducing the time increment and observing variations in results.

Recurrence Formula. Specifically, the following general recurrence formula for all interior nodes in the Somogyi model was derived from equation 3.23 by approximating the time derivatives with forward differences (see Figure 4.3), the derivatives respect to space by central differences, and by replacing the values of void ratio, effective stress, and permeability at the time of analysis with the previous time step values

$$S_{i,j}\delta(K_{i,j}+D_{i,j})u_{i+1,j+1} + (1-2S_{i,j}K_{i,j}\delta)u_{i,j+1} + S_{i,j}\delta(K_{i,j}-D_{i,j})u_{i-1,j+1} = u_{i,j} + \delta_b\gamma z \tag{4.1}$$

where

$$S_{i,j} = \frac{\sigma'^{\beta}}{\alpha}{}_{i,j}$$

$$K_{i,j} = \frac{k_{i,j}}{1+e_{i,j}}$$

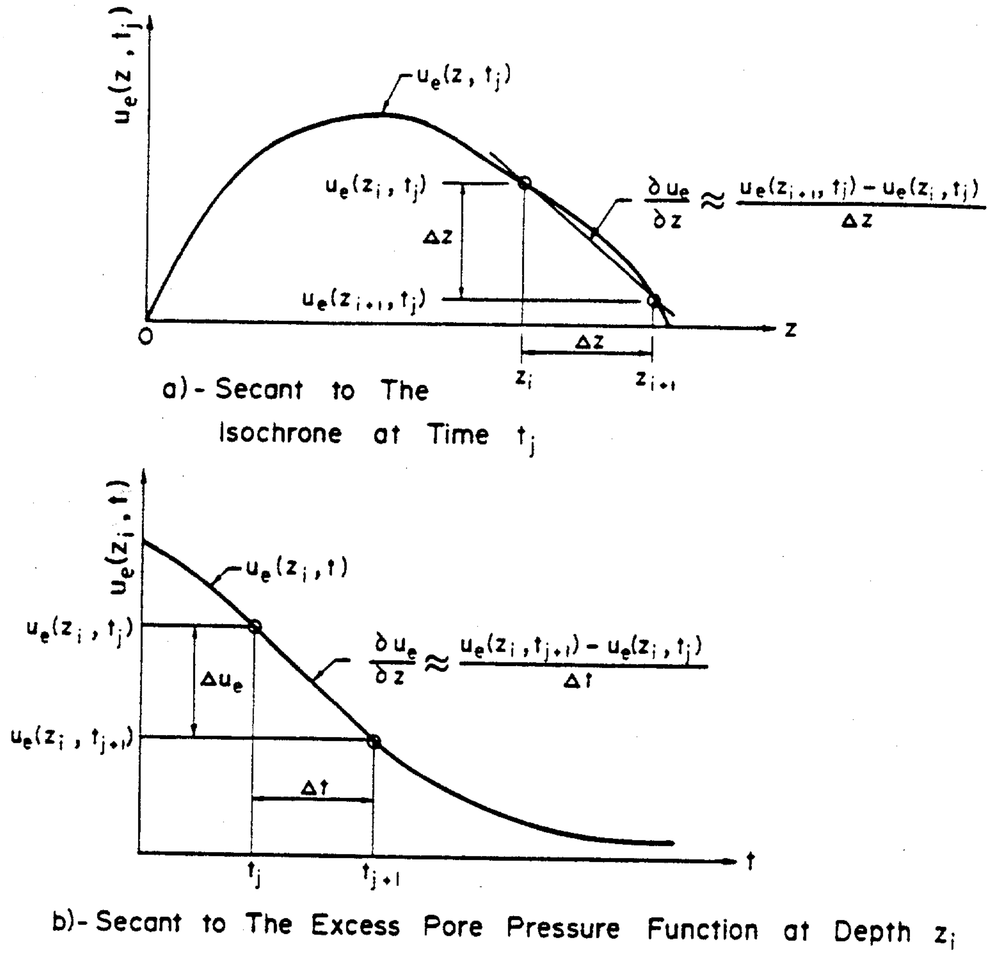

Figure 4.3. Definition of Partial Derivatives in terms of Forward Finite Differences (Perloff and Baron, 1976)

$$D_{i,j} = \frac{1}{4}\left(\frac{k_{i+1,j}}{1+e_{i+1,j}} - \frac{k_{i-1,j}}{1+e_{i-1,j}} \right)$$

$$\delta = \frac{\Delta t}{(\Delta z)^2}$$

Δt = time increment

Δz = space increment

Boundary and initial conditions need to be specified for a complete solution. If the lower boundary (i=1) is impermeable (single drainage)

$$\frac{\partial u}{\partial z} = 0. \tag{4.2}$$

This is known as a reflecting boundary, since in terms of central differences, it implies that

$$u_{i+1} = u_{i-1} \tag{4.3}$$

Substituting equation 4.3 into 4.1, the following boundary recurrence formula is obtained for single drainage

$$2S_{1,j}K_{1,j}\delta u_{2,j+1} + (1-2S_{1,j}K_{1,j}\delta u)u_{1,j+1} = u_{1,j} + \gamma_b\Delta z \tag{4.4}$$

At the upper boundary, the excess pore pressure is always equal to zero. Substituting equation 4.4 into equation 4.1, the following recurrence formula is obtained for the node just below the surface

$$u_{N-1,j+1} + S_{N-1,j}K_{N-1,j}\delta(-2u_{N-1,j+1} + u_{N-2,j+1})$$

$$S_{N-1,j}\delta(K_{N-1,j} - D^*_{N-1,j})u_{N-2,j+1} = \gamma_b\Delta z \tag{4.5}$$

where

$$D^*_{N-1,j} = \frac{1}{4}(K_{N,j} - K_{N-2,j})$$

In solving equations 4.1, 4.4, and 4.5 simultaneously, the excess pore pressures are known as time progresses, and the effective stresses can then be readily obtainable from the following equation

$$\sigma'(z) = \sigma'_o + \sigma_b(z) - u(z) \qquad (4.6)$$

where

σ'_o = initial effective stress

From the effective stress, the corresponding void ratios, solids contents, and permeabilities are calculated, and the solution proceeds to the next time step. It should be noted that for double drainage, the lower boundary condition is given by u(O,t) = 0.

Sand-Clay Mixes. A peculiarity inherent in this analysis is that in predicting the behavior of sand-clay mixtures the Somogyi model assumes that the compressibility and permeability of the mixture are controlled by the properties of the clay and are independent of the presence of sand, which merely acts to increase the unit weight. In essence, this assumption implies that all the water in the mixture is associated with the clay particles, with each sand particle floating in the clay-water matrix. In the case of Florida phosphatic clay wastes, this assumption appears to be valid for sand-clay ratios up to five and clay solids contents up to 60%, which encompasses the normal range of applications (Somogyi et al., 1981).

Description of the Prosrams. Two sets of four computer programs (single drainage and double drainage) were developed. FLINS is used to analyze densification during an initial filling period. FLCOS handles the problem of densification during filling over an existing deposit. This program is utilized whenever a change in filling rate occurs or when filling is resumed after a period of quiescent settling. In these cases, continuity at the interface between the new and old deposits is ensured by the fact that the solids content at the surface of the existing deposit is identical to that attained by the new material immediately after deposition. The filling programs perform analyses for a specific time period or until a maximum deposit height is reached. QSNS is used to analyze consolidation during quiescent settling of a clay deposit whose solids content varies with depth, and is used when filling has been interrupted, either temporarily or permanently. Finally, QSUS is used for analysis of self weight consolidation during quiescent settling of a clay deposit whose solid content is uniform with depth.

The Somogyi programs require the following input: 1) compressibility and permeability of the sediment; 2) initial void ratio of the sediment; 3) size of the disposal area; 4) filling rate of dry solids; 5) filling period; 6) surcharge (if any); and 7) boundary drainage conditions. The output of the programs includes: 1) settlement vs. time: 2) void ratio vs. depth and time; and 3) pore pressure vs. depth and time.

The output of one program becomes the input for the next program and virtually any sequence Of filling and quiescent consolidation can be analyzed.

Yong Model

General. Yong chose an explicit finite difference scheme to solve equation 3.27. This method allows for easy manipulation of the governing equation, but has stability and convergence restrictions which will be discussed herein.

Recurrence Formula. As presented in chapter 3, Equation 3.27 represents the governing equation for this model. By taking the three representative horizontal planes with unit cross-sectional areas shown in Figure 4.4, separated by a discrete distance Δz, and a second intermediated set of planes (1,2) equally spaced within the primary planes, and substituting the first derivatives respect to space with forward differences (see figure 4.3), and by substituting the second derivative respect to space with the central difference

$$\frac{\partial^2 u}{\partial z^2} = \frac{u(p-1) - 2u(p) + u(p+1)}{\Delta z^2}$$

and rearranging, the following general recurrence formula computing the change in excess pore pressure as time progresses is obtained

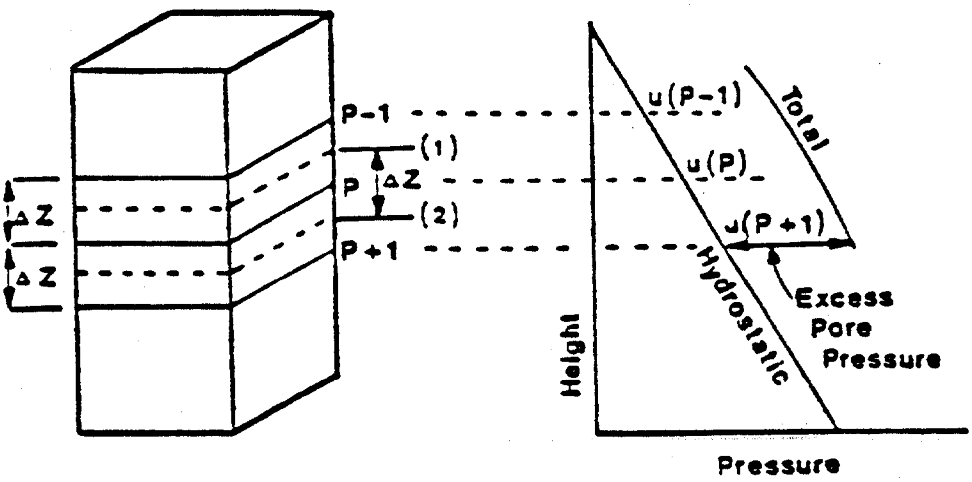

Figure 4.4. Discretization of Elemental Volume Used in the Derivation of the UF-McGS, UF-McGM and Yong Models (Yong et al., 1983).

$$\Delta u(p) = \frac{(1+e)\Delta t}{\gamma_w a_v \Delta z} [K_1(u(p)-u(p-1))-K_2(u(p+1)-u(p))] \quad (4.7)$$

Equation 4.7 drives the process of consolidation by explicitly updating the excess pore pressures. However, in order to maintain non-linearity, not only need the excess pore pressure be updated, but also the material parameters (void ratio, effective stress, permeability) and, most importantly, the geometry of the impoundment. This is crucial, since the analysis is performed in respect to convective coordinates; therefore, if the void ratio is updated, the convective coordinates must be converted into updated Lagrangian coordinates to be consistent. Material property updating is performed by logarithmic interpolation, and pond geometry is updated by linear interpolation. Figure 4.5 shows the computational flow scheme for the Yong model.

Continuous Fill. Continuous fill is modelled differently in the Somogyiand Yong models. While Somogyi accounts for continuous fill implicitly by computing the change in buoyant stress with time ($D\sigma_b/Dt$) in the governing recurrence formula which updates the pore pressures, Yong considers continuous fill explicitly by not including the change in buoyant stress term ($D\sigma_b/Dt$) in the governing recurrence formula, but instead by adjusting the pond height due to fill occurring within every iteration and by recomputing the pore pressures obtained from the governing recurrence formula at the beginning of each time step, rearranging equation 3.15. Thus, the σ_b term in

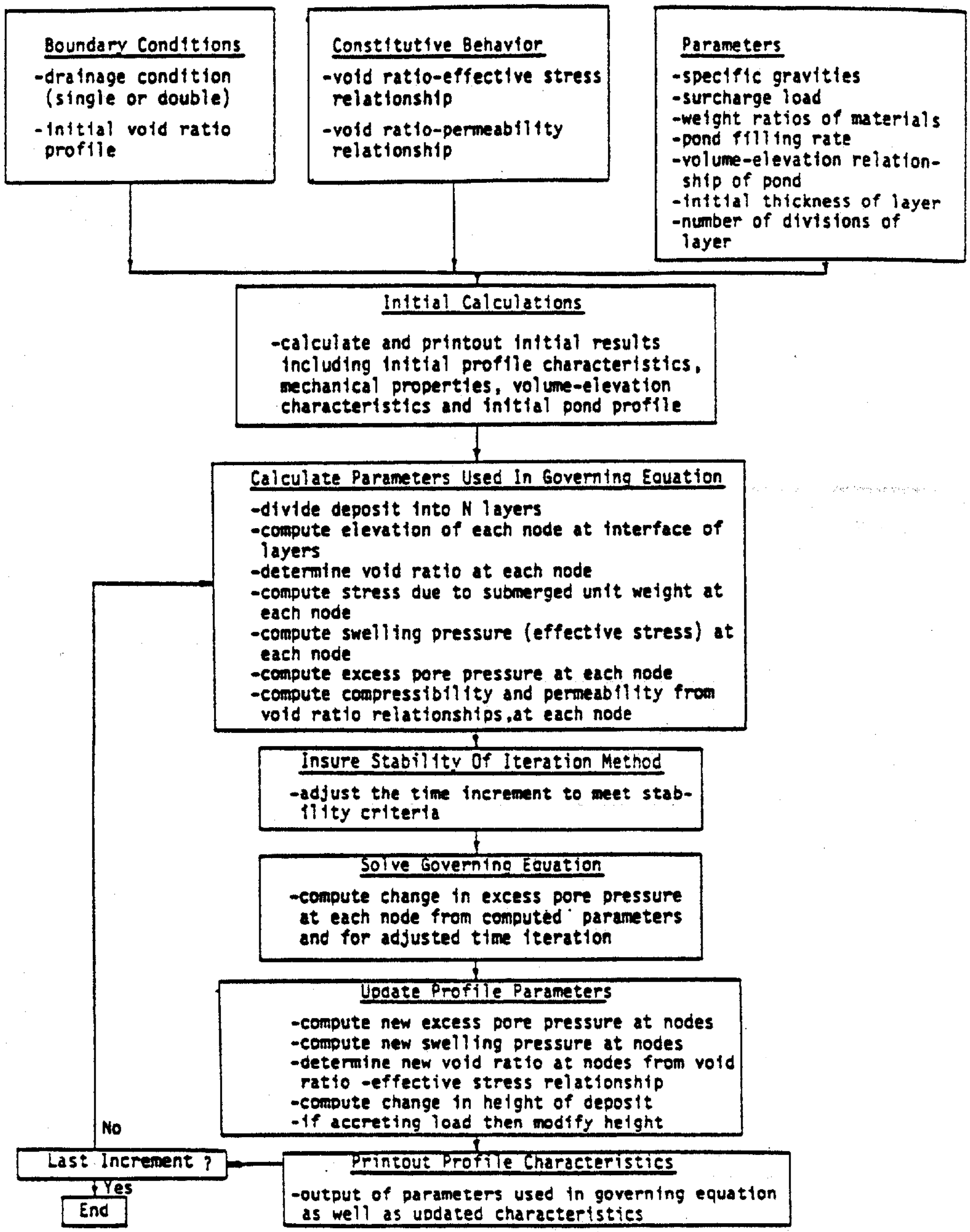

Figure 4.5. Computational Flow Scheme of Yong's Model (Yong and Ludwig, 1984).

this equation is increased by an amount corresponding to the change in buoyant stress due to continuous fill occurring in the previous time step.

Stability and Convergence. Since the governing equation for this model is an expansion of a parabolic partial differential equation, an explicit solution is numerically stable only if

$$0 < \alpha = c_v \frac{\Delta t}{(\Delta x)^2} < 0.5 \qquad (4.8)$$

where

Δx = layer thickness

However, Ames (1969) points out that for $\alpha = .16$, the truncation error for the explicit method goes to zero; therefore, running the program with α fixed at .16, allows for rapid convergence. However, this may not produce accurate solutions if the time steps generated are too extensive, since the program assumes a linear void ratio within each iteration. Stability is checked in this model by calculating the minimum Δt throughout the individual layers based on the corresponding c_v values. If the minimum Δt violates the stability criteria, the time step is shortened to accommodate the α restriction. Herein lies a drawback for this model since the nature of the solution technique is tedious and requires enormous computational efforts for large and soft deposits (high initial void ratios).

Initial and Boundary Conditions. The Yong model allows for both single and double drainage. For single drainage, as with the Somogyi model, equation 4.2 and the reflecting boundary concept apply: therefore, the governing recurrence formula for the bottom node must be adjusted accordingly, thus

$$\Delta u(np) = 2\frac{(1+e(np))\Delta t}{\gamma_w a_v \Delta z}[K_n(u(np)-u(n))] \tag{4.9}$$

where

np = bottom boundary

n = node above bottom boundary

For double drainage,both the top and bottom boundaries are assumed to be free-draining and developing no excess pore pressure. Therefore, the u(np) = 0.

For both single and double drainage, the void ratio at the upper boundary is fixed at the original in situ value prior to consolidation. This constitutes the initial value condition (Yong and Ludwig, 1984).

Initial Void Ratio-"Initial Effective Stress"

According to the effective stress-void ratio relationship established for each particular impoundment by the Somogyi and Yong programs, every void ratio has a corresponding effective stress. Therefore, according to this criteria,if a pond has an initial void ratio, then it must have a corresponding effective stress. Physically, however, self-weight consolidation occurs as a result of the dissipation of excess pore pressures, which generate corresponding effective stresses. Thus, initially, if no consolidation has taken place, no effective stress should be present. This apparently presents a dilemma, and a possible shortcoming in the Somogyi, UF-McGS, and Yong programs, since they assume that the pond has an initial effective stress corresponding to the initial void ratio of the pond, in order to satisfy material relationships and begin the analysis. However, the dependent variable for which a solution is obtained in the governing equations of these programs is the excess pore pressure, not the effective stress or void ratio. Initially, then, the excess pore pressure should be zero at the top of the pond and should increase by an amount equal to the buoyant stress throughout the cross-section of the impoundment. Since the initial excess pore pressure distribution is calculated by subtracting the initial effective stress from the initial buoyant stress, the programs account for the initial effective stress by increasing the initial bouyant stress by an amount equal to the initial effective stress. Figure 4.6 shows a model pond 10 ft. high, with a bouyant weight of 10 pcf and an initial void ratio which corresponds to an initial effective stress of 10 psf. (A) shows the calculation of excess pore pressure as observed in the field, while (B) shows the computer program calculation of excess pore pressure.

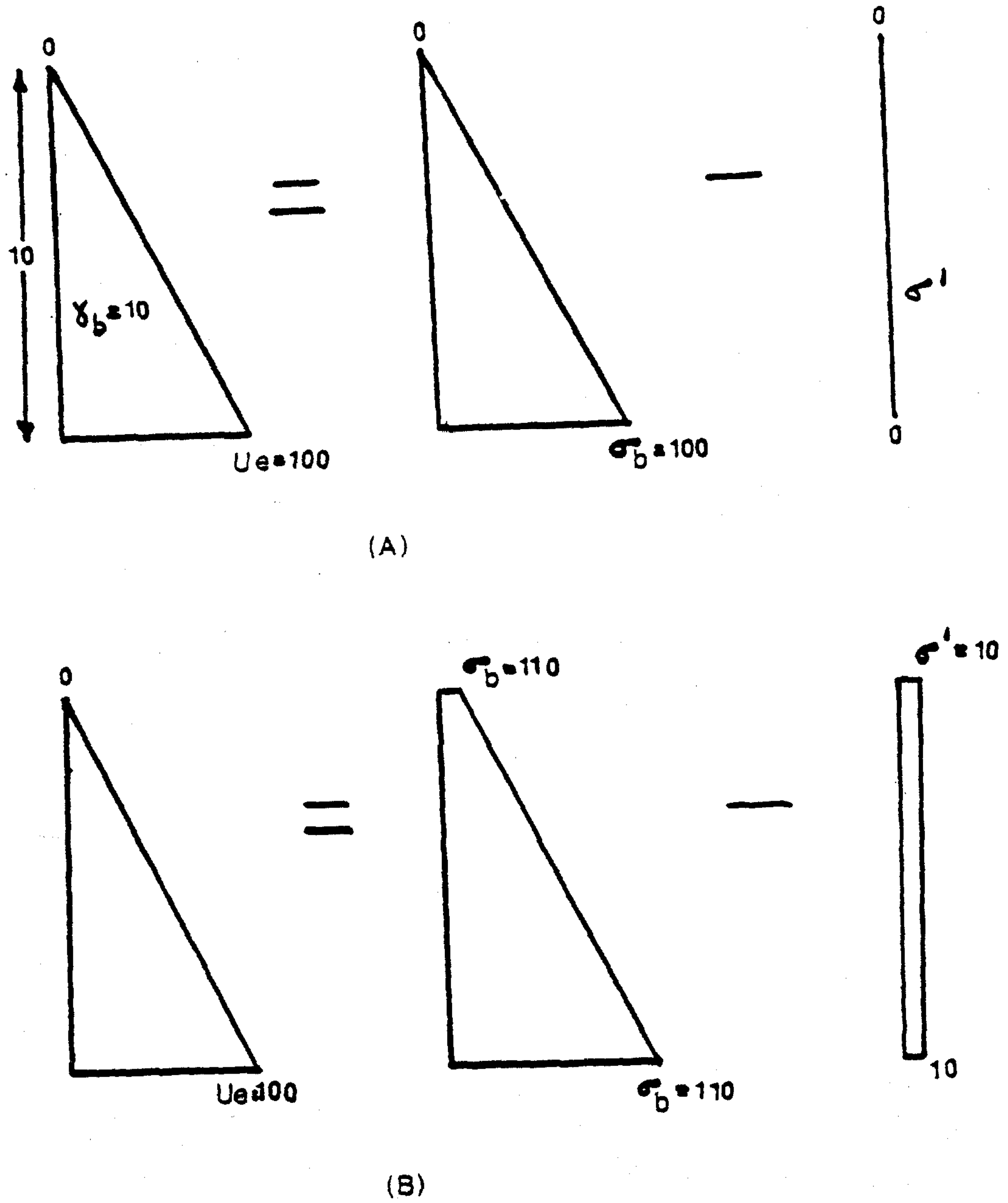

Figure 4.6. Calculation of Initial Excess Pore Pressure.
(A) Actual Field Condition
(B) Somogyi and Piecewise Linear Programs

CHAPTER FIVE
UF-McG MODELS

Introduction

This chapter will discuss in detail the development of the UF-McGS (single layer) program, and the UF-McGM (multiple layer) model. Both these models are expansions of the Yong program as documented by Hernandez (1985).

UF-McGS Model

General

As noted in Chapter 1, the Yong model allows consideration of non-homogeneous profiles, and thus provides a viable tool in modelling large strain consolidation. However, the following drawbacks limit its applicability:

1) Initial filling conditions are difficult to model.
2) When continuously filling, the new material added must have the same void ratio as the underlying material.

Also, the following characteristics of the program limit its versatility:

1) Only allows individual data point input of the material parameters, instead of a power function.
2) Data can only be inputted in the metric unit system.

3) Definition of void ratio in terms of clays, non clays, bitumen, and organics.

4) Only one filling or the quiescent condition may be modelled.

5) Input parameters can only be inputted in batch mode,

6) No graphics capabilities.

7) Output of results is cumbersome.

The first objective pursued in this research was to generalize the data input to accommodate any large strain consolidation problem. In fulfilling this purpose, interactive and batch input of data and a built-in batch file editor were developed. This built-in preprocessor allows the user to perform parameteric studies without major difficulty. After the preprocessor was completed, the following major changes and additions, which will be discussed individually, were made to the available Yang code:

1) Option allowing for automatic generation of power curves or manual point by point definition of material properties.

2) Modification to the formula calculating the void ratio and bouyant weight (see Appendix A).

3) Modification to the initial filling condition.

4) Modification of pore pressure updating for continuous filling conditions.

5) Modification of the method for checking stability.

6) Improved calculation for the average degree of consolidation.

7) Addition of Lotus 1-2-3 compatability for graphics.

8) Consideration of sand-clay mixes

Input of Material Parameters

As discussed previously in the description of the Somogyi model, several researchers (Keshian et al., 1976; Roma, 1976; Wissa et al., 1983) have determined that a power curve can be used to describe best the relationship between the material properties (void ratio-effective stress-permeability) for mineral slurries. Thus, the UF-McGS model allows for automatic generation of power curves, once the A,B,C, and D parameters (equations 3.24 and 3.25) are inputted. However, if field tests are performed that determine that the best fit between the material parameters is not a power curve, the UF-McGS model permits manual point by point description of these relationships, which are joined by line segments between the points.

Initial Filling Condition

Since the updating of pore pressures for the Yang model is based upon previous pond conditions, determination of consolidation during initial filling becomes impossible unless an initial pond height is assumed. This criteria was standardized by the UF-McGS model by creating an initial condition which varies depending upon the filling characteristics of each pond. The initial condition assumes 1 day of filling where no consolidation occurs, and thus an initial triangular excess pore pressure distribution is created. This length of time was picked arbitrarily such that the amount of consolidation occurring in that span of time could be considered negligible, and such that the height of fill incorporated during the time span could be negligible compared to subsequent filling.

To determine the progress of consolidation, this initial height is divided into layers with a uniform void ratio distribution determined by the solids content at which the material is being filled. Then, the effective stress and permeability are obtained from the corresponding constitutive relationships, compressibility calculations are performed, and the excess pore pressures are updated by adding the change in buoyant stress occurring in the previous time step (γ_b x height of fill added during the iteration) to equations 4.8 and 4.10, respectively. Therefore, the upper node has an excess pore pressure equal to the change in buoyant stress. The geometry is then updated by adding the height of fill inherent to the previous time step, and the pore pressures interpolated so that they correspond to the redefined node positions and the upper node has no excess pore pressure.

During the first iteration in initial continuous filling, the $\Delta\sigma_b/\Delta t$ term is zero, because explicit recurrence formulas only consider the change in buoyant stress inherent to the previous time iteration. However, subsequent time steps do consider the $\Delta\sigma_b/\Delta t$ term. It should be noted that the $\Delta\sigma_b/\Delta t$ term is set to zero for quiescent consolidation.

Interpolation of Pore Pressures in Continuous Fill

As discussed in the previous section, the UF-McGS model does not apply equation 3.15 to update pore pressures as the Yong model does, but instead interpolates the values obtained from a governing recurrence formula which includes the change in buoyant stress with time.

Figure 5.1 shows a comparison between both approaches, and the Somogyi model for pore pressure distribution at 0.5 years for a model pond (3a), to be discussed in the next chapter. Results indicate that at the same number of layers, the interpolation approach appears to agree with the Somogyi model better than does the Yong approach. However, there is a 2.7 % difference between the UF-McGS model and the Somogyi model at the bottom boundary; this is reduced to 2.3 % when performing a 60 layers UF-McGS analysis. This may be due to their different approaches when modelling continuous fill. While UF-McGS keeps the number of layers constant throughout the analysis, the Somogyi model adds a new layer for each time step during continuous filling. The latter approach is more accurate because UF-McGS interpolates values; however, when analyzing ponds with intermittent fill, by constantly adding layers, the Somogyi

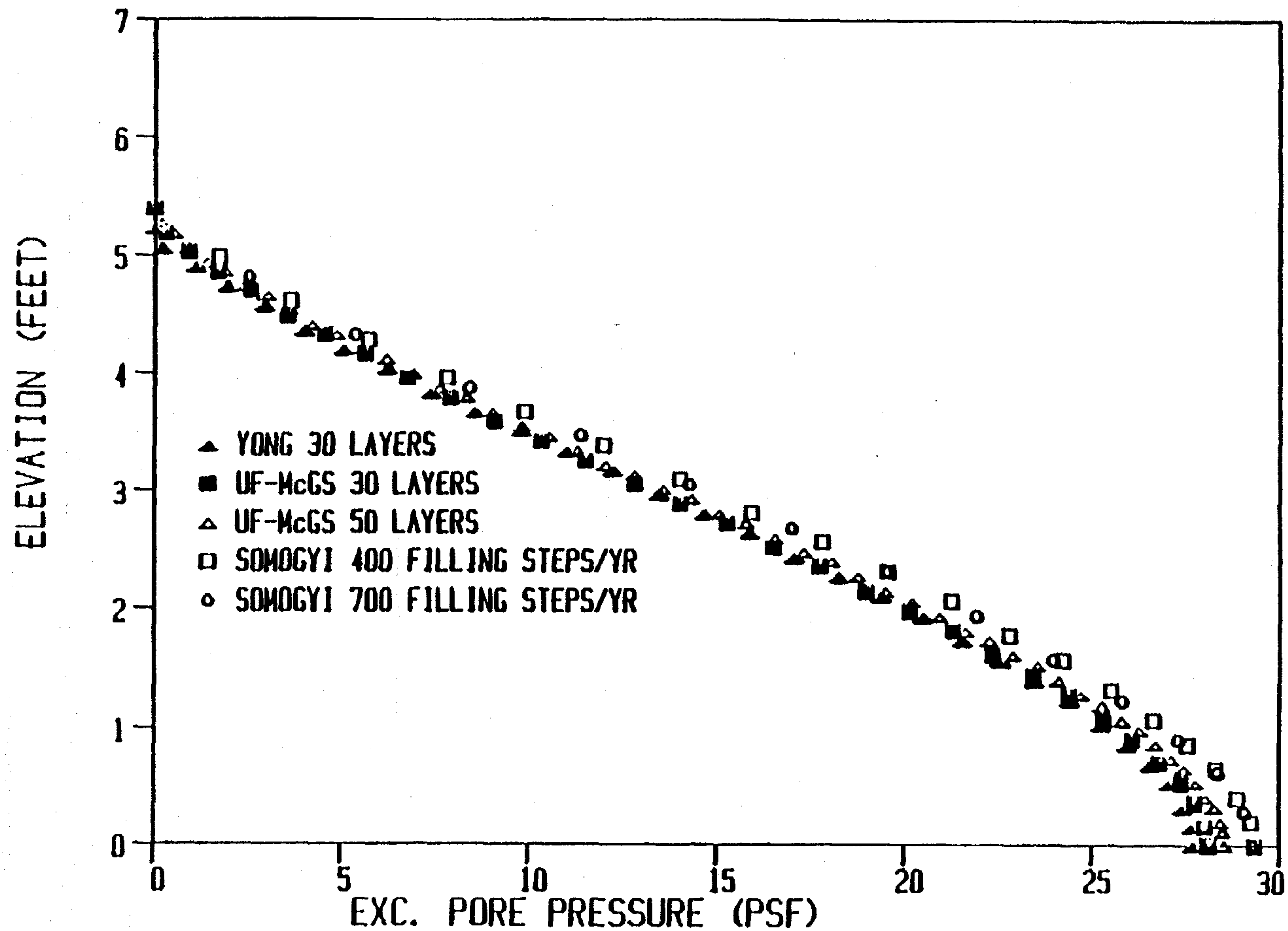

Figure 5.1. Pore Pressure Distribution at Pond 3a at .5 years.

program becomes very slow. It should be noted that although this pore pressure difference exists, the rate of consolidation and the height of pond agree very well for both models in continuous filling.

Stability and Convergence

Although Yong and Ludwig (1984) discuss that the Yong model maintains an α value between 0.16 and 0.5, the code documented by Hernandez (1985) allows for values of α less than .16, based on the minimum c_v of the deposit. This seems reasonable for thick ponds, because by not allowing α to assume values lower than .16, the initial time steps generated may be in the order of weeks.

In considering stability and convergence, the UF-McGS model follows one approach for initial filling and a different approach for intermittent stage filling-quiescent consolidation. For initial filling conditions, since the assumed initial filling condition creates very small space steps (1 day of filling), very small time steps are generated at the beginning of the analysis. Thus, it is necessary to increase the value of α to maximize the size of the time step. The UF-McGS model fixes α at .45 during initial filling unless the time step becomes greater than 300,000 seconds (3.47 days), in which instance the α value is reduced accordingly. For intermittent stage filling-quiescent consolidation, the problem is the opposite, where due to the thickness of the deposit, time steps may be too extensive and updating of pond non-linearity may be inaccurate. Therefore, to allow for rapid convergence, the a value is fixed at .16 unless the time step becomes greater than 300,000 seconds, in which case thea value is reduced accordingly. The value of 300,000 seconds was selected arbitrarily to maintain approximately 10 time steps/month. Parametric studies comparing a fixed a value of .16 throughout the analysis and a value fluctuating between 0 and .16 are shown in the next Chapter. It should be noted that when an impoundment is divided into more than 15 layers, the a value is fixed at .45 in order to increase time steps, unless the value of 300,000 seconds is exceeded.

Improved Calculation for the Average Decree of Consolidation

The average degree of consolidation U(t) is defined as

$$U(t) = \frac{\rho c(t)}{\rho c} \tag{5.1}$$

where

$\rho c(t)$ = the consolidation settlement at time t.

ρc = the ultimate value of the consolidation settlement.

Equation 5.1 may be expressed in terms of the void ratio change by the following expression

$$U(t) = \frac{\int [e(\xi,0) - e(\xi,t)]d\xi}{\int [e(\xi,0) - e(\xi,\infty)]d\xi} \tag{5.2}$$

According to Perloff and Baron (1976), by assuming the coefficient of compressibility a_v to be constant for the range of effective stresses occurring at a point, the changes in void ratio may be expressed as

$$e(\xi,0) - e(\xi,t) = a_v[u(\xi,0) - u(\xi,t)]$$
$$e(\xi,0) - e(\xi,\infty) = a_v[u(\xi,0) - u(\xi,\infty)]$$

Substituting the above equations into equation 5.2, and assuming further that a_v is constant throughout the stratum, then it may be removed from within the integral and the average degree of consolidation may be expressed as

$$U(t) = 1 - \frac{\int [u(\xi,t)d\xi}{\int [u(\xi,0)d\xi} \tag{5.3}$$

Equation 5.3 is used by Yong to calculate the average degree of consolidation. However, since a_v is not constant throughout the progress of large strain consolidation, equation 5.2 appears to be a more proper way of calculating this parameter and is thus used for the UF-McGS model. In order to apply equation 5.2, three steps must be followed:

1) calculation of $e(\xi,0)$

2) calculation of $e(\xi,\infty)$

3) Integration of denominator and numerator throughout the pond profile

The calculation of the void ratio at infinity for any node is simple because by adding the excess pore pressure to the existent effective stress, the final effective stress is obtained. Knowing this effective stress, the final void ratio is obtained by applying the corresponding constitutive relationship between the void ratio and effective stress.

On the other hand, the determination of the initial void ratio at a node is more difficult because by using an updated lagrangian scheme, the position of the nodes changes with time, and must be considered differently for quiescent and continuous filling. For quiescent consolidation, it is necessary to convert the initial and existing geometry of the pond into reduced coordinates (reduced coordinates do not change with time) in order to find the corresponding initial void ratio for an updated lagrangian position. However, during continuous filling, reduced coordinates change with time due to the volume of solids added. Therefore, in order to relate the void ratio at an updated node to the corresponding initial void ratio, a profile of the height of solids to be filled in the pond with corresponding initial void ratios must be calculated and then compared to corresponding updated lagrangian positions.

Addition of Lotus l-2-3 Compatabilitv of Graphics

The two output files created by the user when running the UF-McGS program, with their respective ".prn" extensions, should be manipulated in the following way to provide graphics compatability:

1) LOTUS 1-2-3 should be inserted in drive A and the output files in drive B.
2) After the LOTUS menu appears, the user must press the return key when the pointer is located over the following words: File, Import, Numbers. Then, when asked for the file name to be imported, the user types the name of the file to be graphed.
3) If the error message is "part of the file missing", the user should hit the escape key. This error arises due to the lack of an end of file character.
4) Then, the usual procedure for graphing in LOTUS should be followed (i.e. press the return key when the pointer is located over the words, graph, type, data ranges, etc.).

Sand-Clay Mixes

In predicting the behavior of sand-clay mixes, the UF-McGS model assumes, as Somogyi did,that the compressibility and permeability of the mixture are controlled by the properties of the clay and to be independent of the presence of sand, which merely acts to increase the unit weight.

Figure 5.2 shows the computational flow scheme of the UF-McGS model.

UF-McGM Model

General

To our knowledge, a numerical approach to model piecewise linear multiple layer (different material parameters in each layer) large strain consolidation has not been developed. Therefore, the purpose of this section is to develop a quiescent multiple layer mathematical model that can be incorporated into the existing UF-McGS program.

In order to convert the UF-McGS program into multiple layers, the following major issues must be taken into consideration:

1) Pore pressure and material parameter discontinuity at the interface between different layers.
2) Initial condition surcharge effect of layers above the one being considered, in determining pore pressures.
3) Layers must be analyzed at the same time step.

Interface Considerations

In order to obtain the boundary values for pore pressure, permeability, effective stress, and void ratio, mathematical approximations must be made.

Assuming isotropy, linearity, and neglecting inertia, Preprocessor

-time of analysis
-drainage condition
-specific gravities
-if filling: area of pond, time spans, filling rate, and discharge solids content.
-define profile material characteristics

Calculate Parameters used In Governing Equation

-divide deposit into N layers
-compute elevation at each node at interface of layers
-interpolate void ratio at each node
-if filling condition existent, interpolate the pore pressures
-compute submerged unit weight at each node
-compute effective stress at each node
-if filling, skip next step
-compute excess pore pressure at each node
-compute compressibility and permeability from void ratio relationships at each node
-if initial fill, compute excess pore pressure for initial condition

Insure Stability of Iterative Method

-adjust the time increment to meet stability criteria

Solve Governing Equation

-compute change in excess pore pressure at each node

Update Profile Parameters

-compute new excess pore pressure at nodes
-compute new effective stress at nodes
-compute new void ratio based on corresponding new effective stress
-update geometry

Outputting of results

At specified times,
-void ratio distribution
-effective stress distribution
-pore pressure distribution

Summary of Results
-elevation
-degree of consolidation
-average solids content

Figure 5.2. Computational Flow Scheme of UF-McGS Model

continuity of the fluid phase at the boundary between two different clays may be expressed as

$$\{n(v_f - v_s)\}_- = \{n(v_f - v_s)\}_+ \tag{5.4}$$

where

$_-$ = bottom side of the interface

$_+$ = top side of the interface

By substituting equation 3.7 into equation 5.4, the following expression may be obtained

$$k_- \frac{\partial u}{\partial \xi} = k_+ \frac{\partial u}{\partial \xi} \tag{5.5}$$

Equation 5.5 serves as the basis for obtaining an interface pore pressure value. However, the permeabilities at the interface are unknown. These may be approximated by considering the permeability at the node immediately above and below the interface and the rate of change of the permeability between these nodes and the interface

$$k_{i-} = k_{i-1} + \frac{\partial k}{\partial \xi_1} \frac{\Delta z_1}{2} \tag{5.6}$$

$$k_{i+} = k_{i+1} + \frac{\partial k}{\partial \xi_2} \frac{\Delta z_2}{2} \tag{5.7}$$

where

i = interface node

i+1 = node above the interface node

i-1 = node below the interface node

Δz_1 = thickness of layer in bottom material

Δz_2 = thickness of layer in top material

Then, by replacing the partial derivative above the interface (+) with a forward finite difference and the partial derivative below the interface (-) with a backward finite difference, in the limit, equations 5.6 and 5.7, may be expressed as

$$k_{i-} = \frac{3k_{i-1} - k_{i-2}}{2} \tag{5.8}$$

$$k_{i+} = \frac{3k_{i+1} - k_{i+2}}{2} \tag{5.9}$$

Finally, substituting equations 5.8 and 5.9 into equation 5.5, the following recurrence formula is obtained for the pore pressure at the interface between two different materials,

$$u_i = \frac{\left[\frac{3k_{i+1} - k_{i+2}}{3k_{i-1} - k_{i-2}}\right] \frac{\Delta z_1}{\Delta z_2} u_{i+1} + u_{i-1}}{1 + \left[\frac{3k_{i+1} - k_{i+2}}{3k_{i-1} - k_{i-2}}\right] \frac{\Delta z_1}{\Delta z_2}} \tag{5.10}$$

Implementation of this interface consideration will be discussed in later sections.

<u>Initial Condition Surcharge Effect</u>

Homogeneous, single drainage, quiescent single layer consolidation, initially has a triangular pore pressure distribution with a value at the bottom equal to the buoyant weight times the height. However, in multiple layer analysis, the upper boundary of a layer (except for the top layer) does not have free drainage; therefore, in one dimensional consolidation this boundary value of pore pressure is equal to sum of the initial buoyant stresses for all the layers above the one considered. This surcharge effect is propagated uniformly throughout the layer being considered, thus shifting the pore pressures at any point within the layer by an amount equal to the sum of the initial buoyant stresses for all the layers above the one being considered.

Length of Time Step

In order to analyze the progress of consolidation in multiple layers, the governing recurrence formula must be applied at the same time step for all layers. However, since time step size in explicit finite difference is dependent upon c_v, layer thickness and the α value, a correlation must be made between these parameters such that the stability criteria may be met.

Rearranging equation 4.9, and equating Δt for adjacent layers, the following equation may be obtained

$$\alpha_1 \frac{\Delta z_1^{\,2}}{c_{v1}} = \alpha_2 \frac{\Delta z_2^{\,2}}{c_{v2}} \qquad (5.11)$$

where

c_{v1} = coefficient of consolidation for bottom material

c_{v2} = coefficient of consolidation for top material

If more than two layers are present, equation 5.11 is expanded for all adjacent layers.

The UF-McGM model applies stability criteria by assuming a convenient arbitrary whole number of layers for the bottom material and adjusting the thickness of layers in the upper materials utilizing equation 5.11, and adjusting α for the top layer, such that a whole number of layers may be obtained.

CHAPTER SIX
PRESENTATION AND DISCUSSION OF RESULTS

General

In order to validate the UF-McGS model, finite strain computer models in terms of reduced coordinates and closed form solutions were used for comparison. At the University of Florida, two finite strain computer programs are available: 1) the Somogyi programs, and 2) the Cargill program. The Somogyi program was chosen for comparison when applicable because this program is more familar to the phosphate industry. Also, as discussed in Chapter 3, the only closed form solutions available for settlement comparison were developed by McVay (1986b), and Townsend (1986b), and will be used for comparison when applicable.

Prediction Scenarios

Eight waste clay ponds were analyzed in order to validate the UF-McGS model. These model ponds were obtained from the prediction session of the "Symposium on Consolidation and Disposal of Phosphatic and Other Waste Clays" held in Lakeland, 1987. Ponds la and lb model quiescent consolidation; ponds 2a and 2b model quiescent consolidation with a surcharge: ponds 3a and 3b model stage filling; and ponds 4a and 4b model two layer quiescent consolidation.

Material Properties

The following effective stress-void ratio-permeability relationships (Townsend, 1986a) were used for ponds 1a, 1b, 2a, 2b, 2c, 3a, and 3b:

$$e = 15.07\ \sigma'^{-.22}$$

$$k = .8304E\text{-}06\ e^{4.65}$$

where

σ' = in psf

k = in ft/day

The following effective stress-void ratio-permeability relationships (Townsend, 1986a) were used for the sand/clay mixes in ponds 4a and 4b:

$$e = 32.5\ \sigma'^{-.24}$$

$$k = .4235E\text{-}06\ e^{4.15}$$

where

σ' = in psf

k = in ft/day

Quiescent Consolidation

Pond 1a. Figure 6.1 presents the cross-section of a

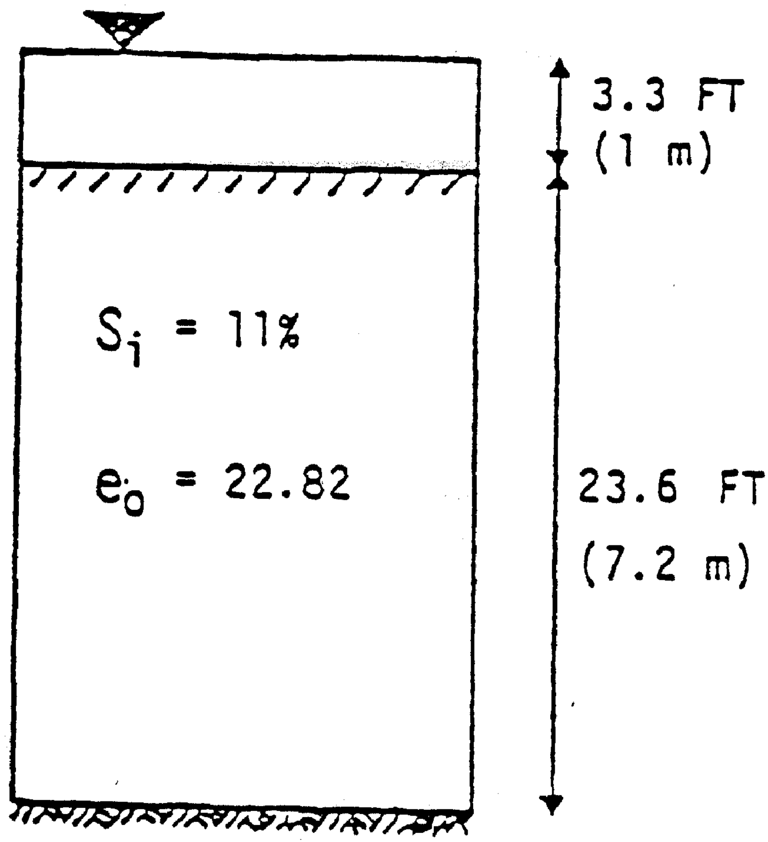

Figure 6.1. Pond 1a: Quiescent Consolidation with Uniform Void Ratio.

23.6 ft deep waste pond with an initial uniform void ratio of 22.82. Figures 6.2, 6.3, and 6.4 show a comparison between QSUS and UF-McGS for the height of pond vs. time, the void ratio, and the excess pore pressure profiles after 1 year of self-weight consolidation, respectively.

Figure 6.2, height vs. time, indicates that the UF-McGS model at 10 layers, 50 points describing the material functions (effective stress, void ratio, and permeability) and α value between 0 and 0.16, has excellent agreement with the Somogyi model at 75 layers, and 100 iterations per year in predicting the progress of consolidation with time. This figure also shows that at 25 layers the Somogyi model slightly over-predicts the most consolidation settlement.

Figure 6.3, void ratio vs. height, displays excellent agreement between the UF-McGS model at 20 layers, 50 points describing the material functions and α between 0 and 0.16, and Somogyi's QSUS at 75 layers and 100 iterations per year. However, the UF-McGS model at 10 layers, and the Somogyi model at 25 layers, diverge from the correct void ratio profile near the top of the pond. The UF-McGS divergence may be due to the fact that this explicit finite difference scheme marches from the bottom up, thus updating the properties near the bottom of the pond faster than those near the top of the pond. The Somogyi divergence is probably due to the coarseness of the mesh.

Figure 6.4, pore pressure vs. height, shows good agreement between the 10 layer UF-McGS analysis and the 50

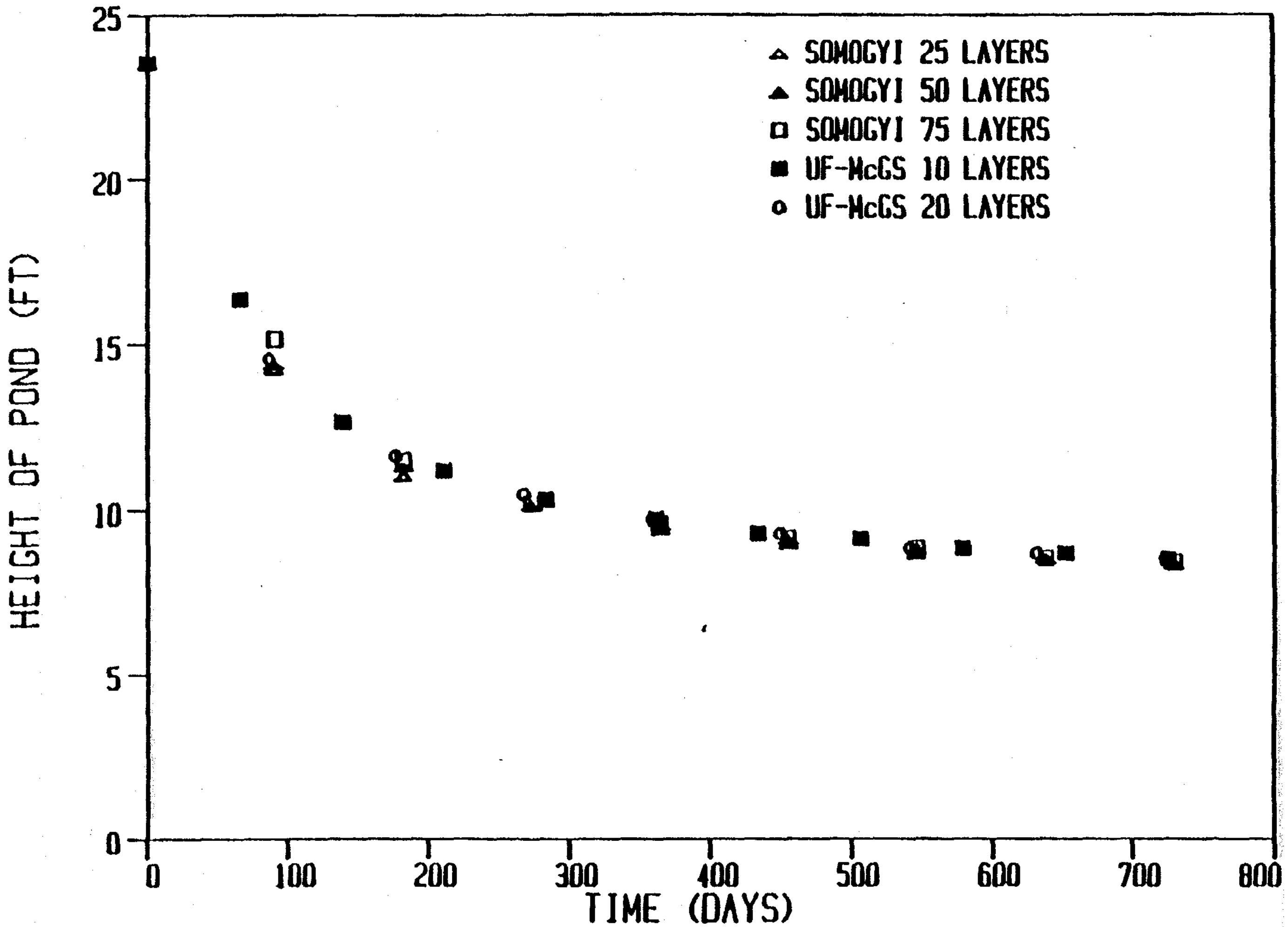

Figure 6.2. Height vs. Time for Pond 1a.

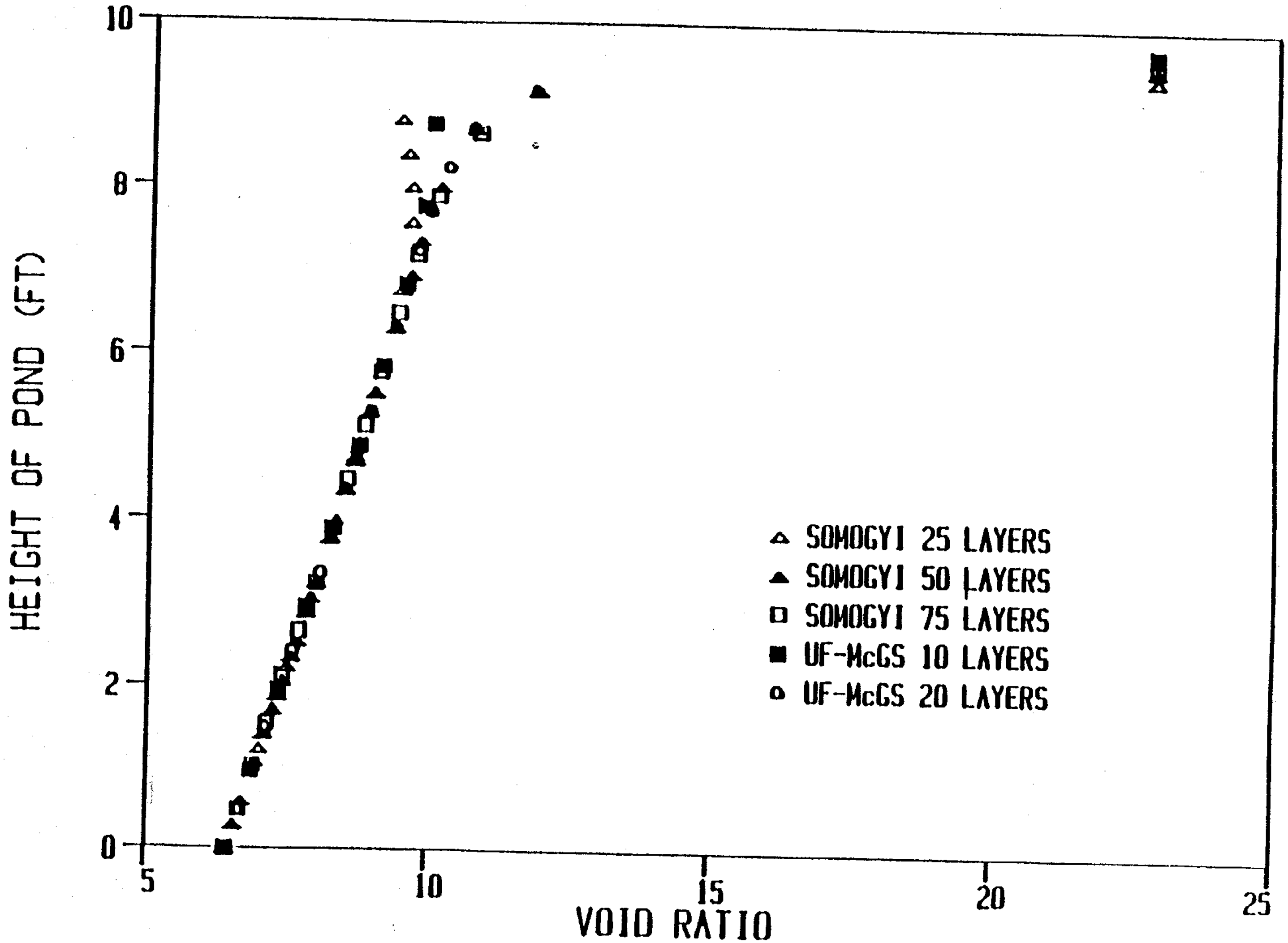

Figure 6.3. Void Ratio Profile after 1 Year for Pond 1a.

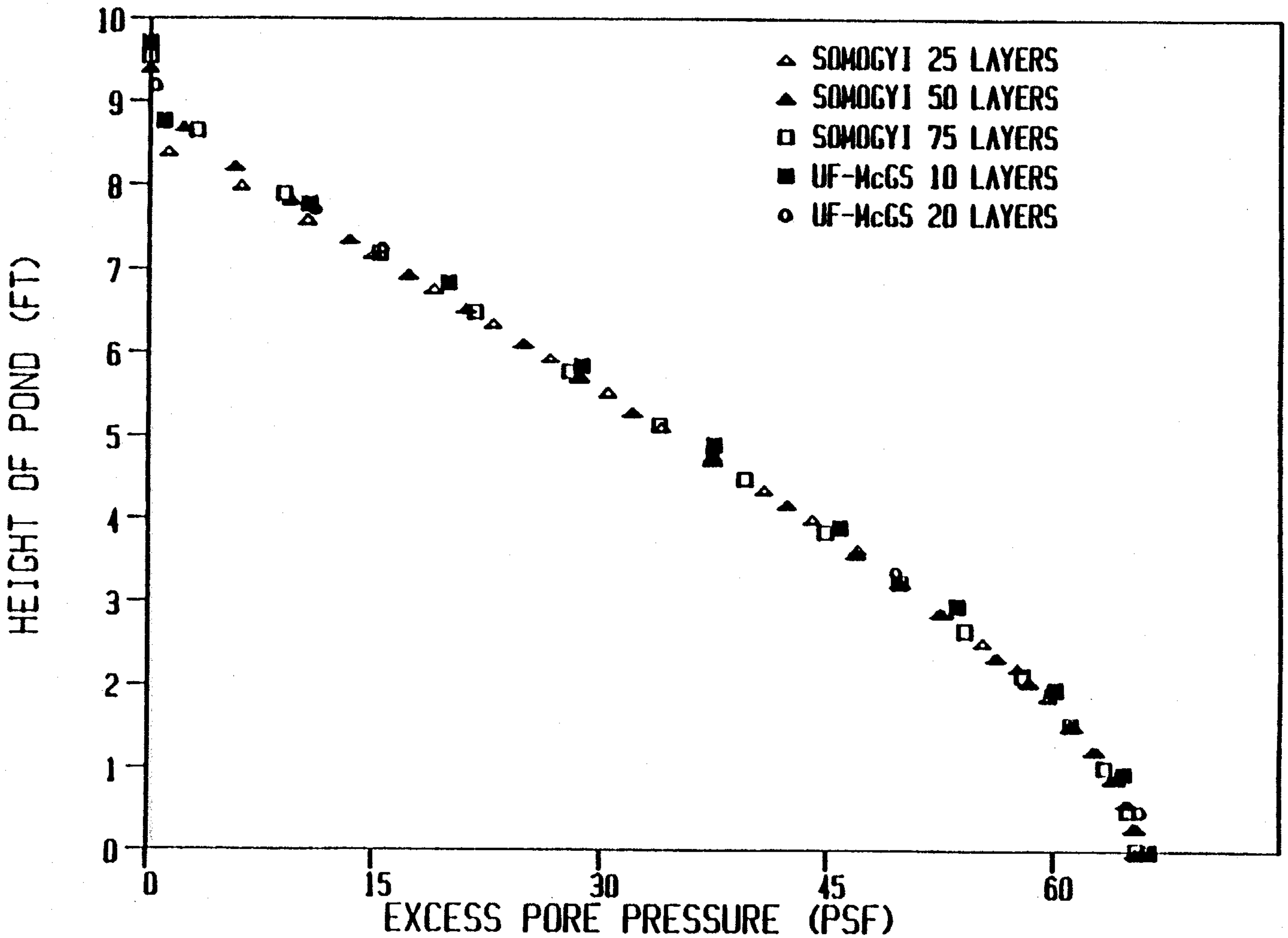

Figure 6.4. Pore Pressure Distribution after 1 Year for Pond 1a.

layer Somogyi analysis. It appears as though the Unrefined 25 layer Somogyi analysis dissipates pore pressures quicker than finer meshes.

Figure 6.5 shows the final void ratio for pond 1. No comparison could be made with the Somogyi model because it does not analyze consolidation after 95% settlement has occured. However, the Somogyi model does predict the final height of pond. This result, as well as the final height obtained from the UF-McGS model and the closed form solution, is presented in Table 6.1, and all show excellent agreement, with less than 1% difference between the predicted final heights.

Figure 6.6 presents the progress of the average degree of consolidation for the Somogyi and UF-McGS models. Excellent correlation is shown, with the 25 layer Somogyi analysis diverging slightly from the other results during the first year of consolidation.

Pond 1b. Figure 6.7 presents the cross-section of a 31.5 ft deep waste pond with an initial uniform void ratio of 14.8. Parametric studies to determine the influence of α in the UF-McGS model and the size of the time steps in the Somogyi model were performed in this pond.

Figures 6.8, 6.9, and 6.10, which present the height of pond vs. time, the void ratio, and the excess pore pressure profiles after 3 years of self-weight consolidation, respectively, show excellent agreement between QSUS at 100 time steps/year and UF-McGS for an a value between 0

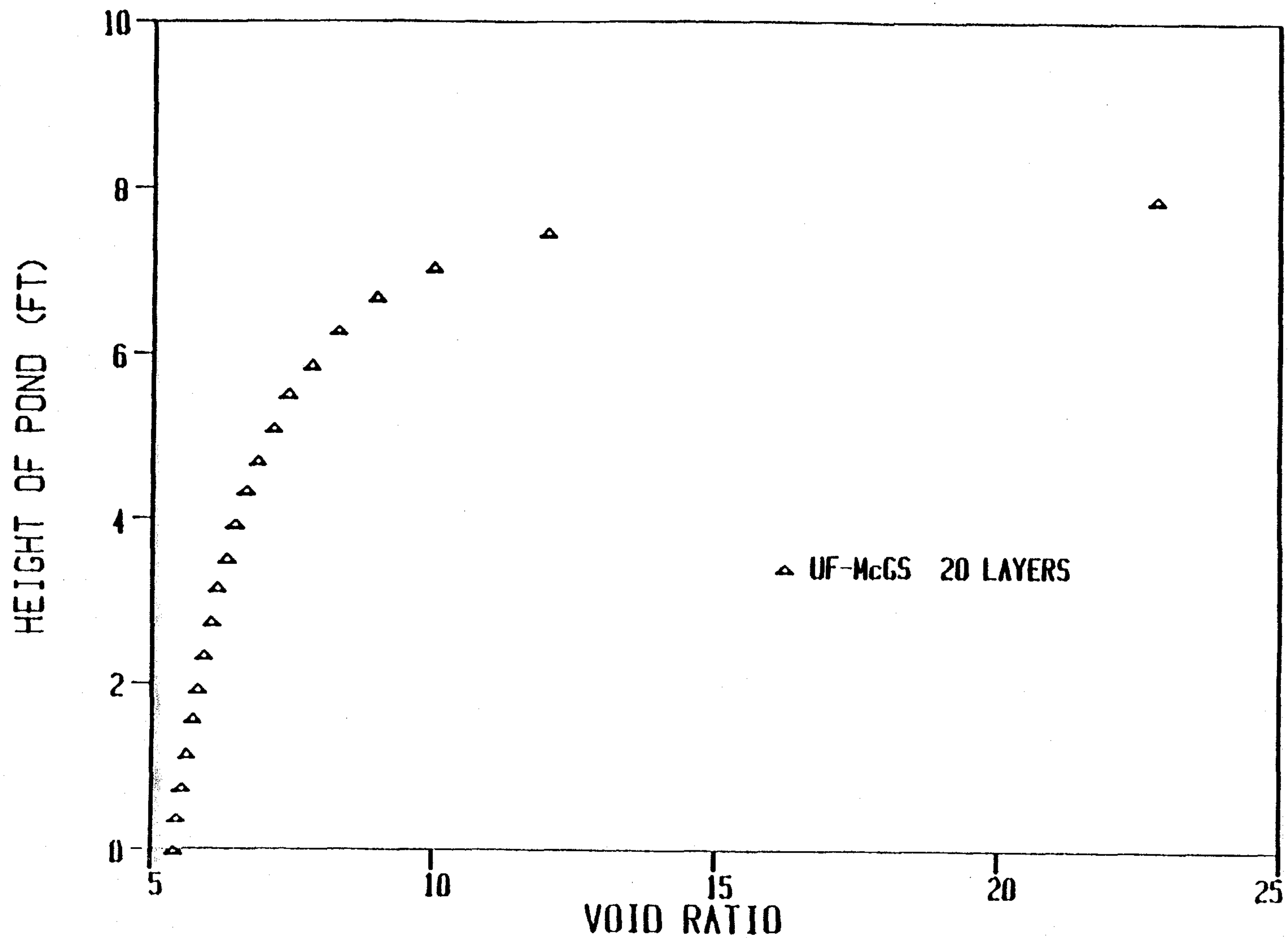

Figure 6.5. Final Void Ratio Distribution for Pond 1a.

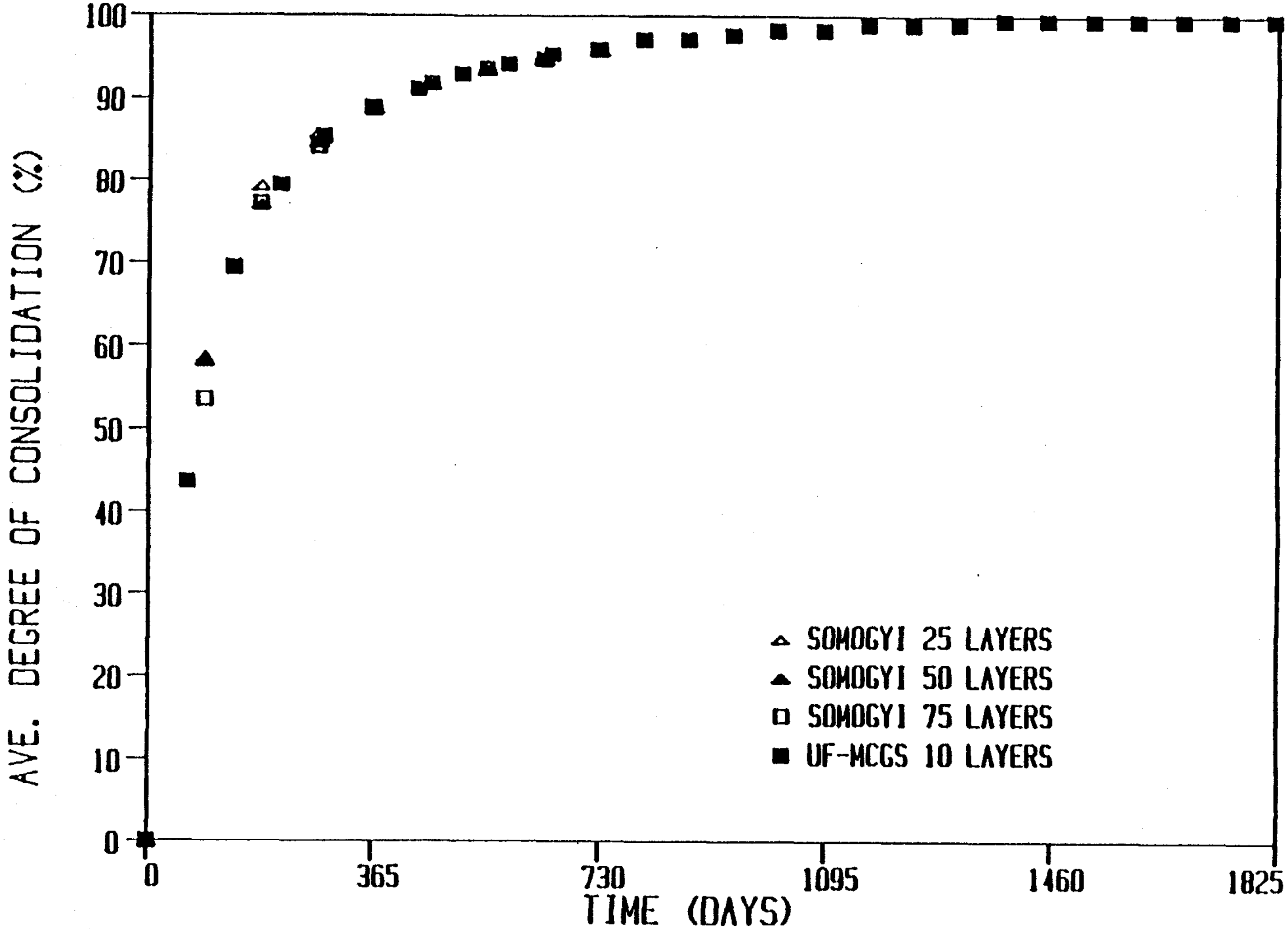

Figure 6.6. Average Degree of Consolidation vs. Time for Pond 1a.

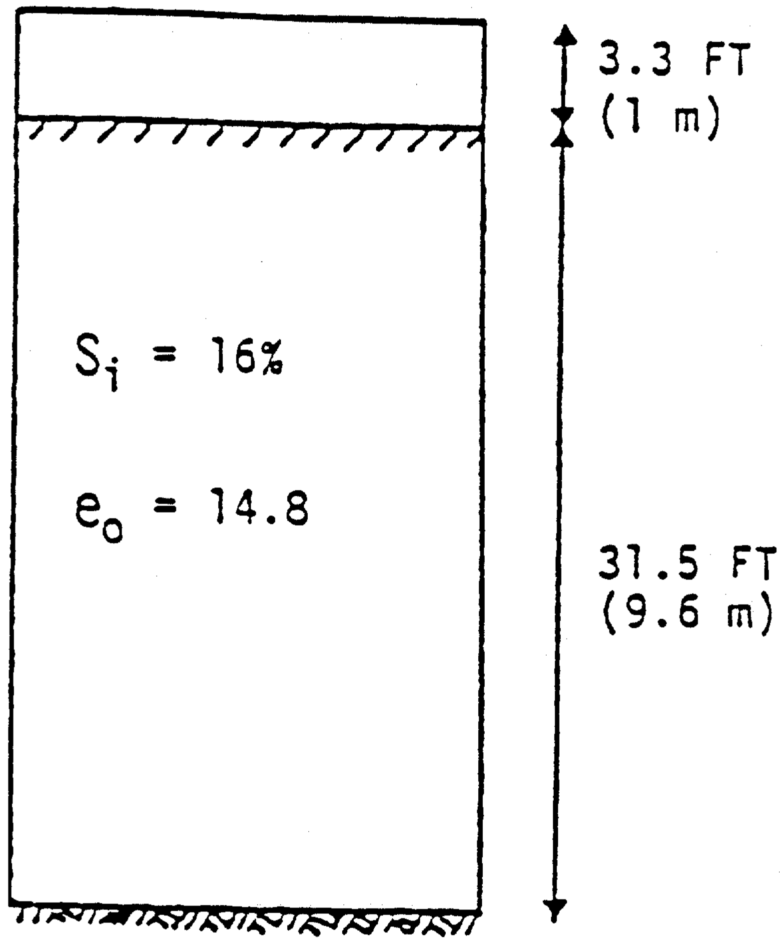

Figure 6.7. Pond 1b: Quiescent Consolidation with Uniform Void Ratio.

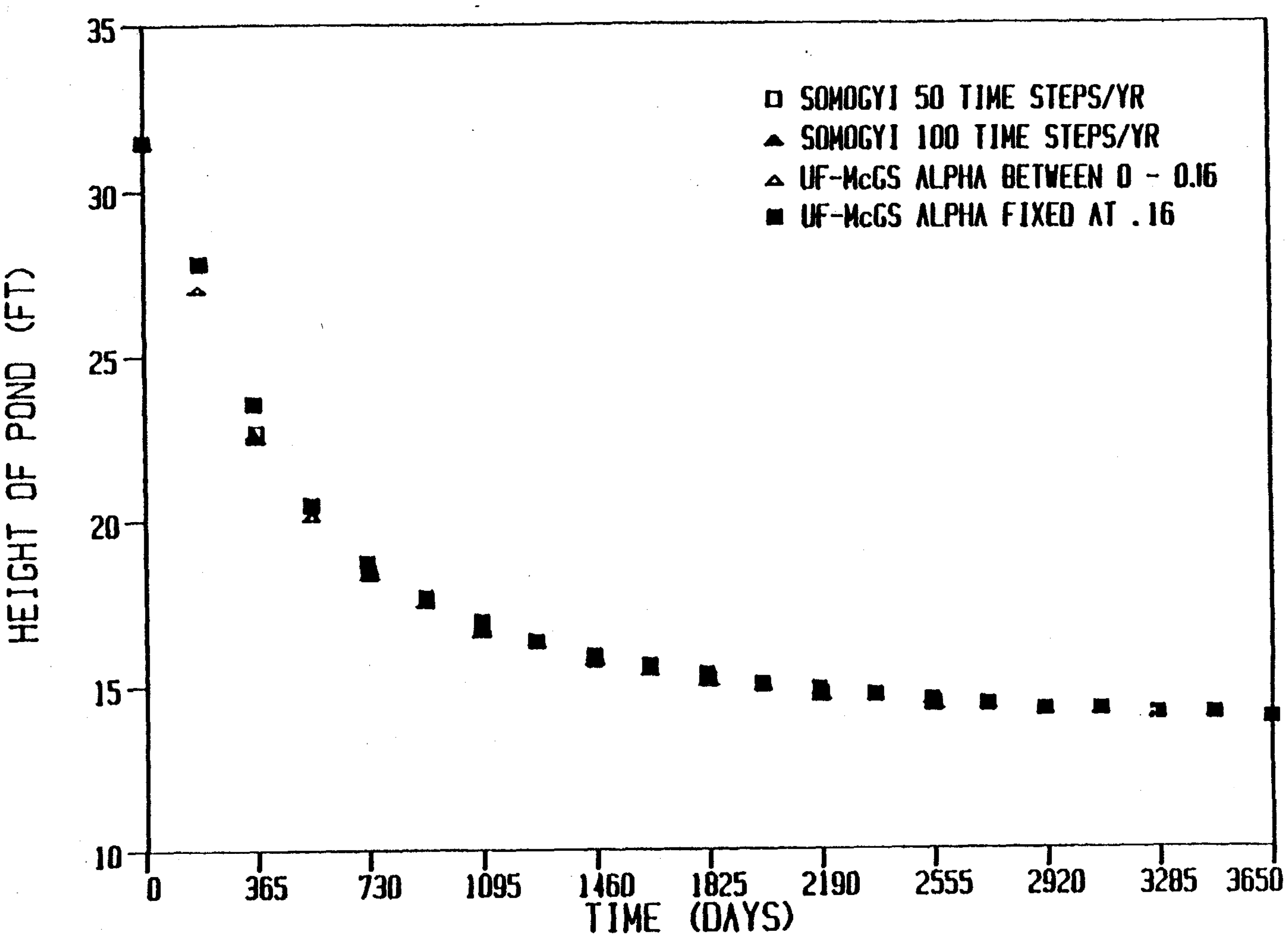

Figure 6.8. Height vs. Time for Pond 1b.

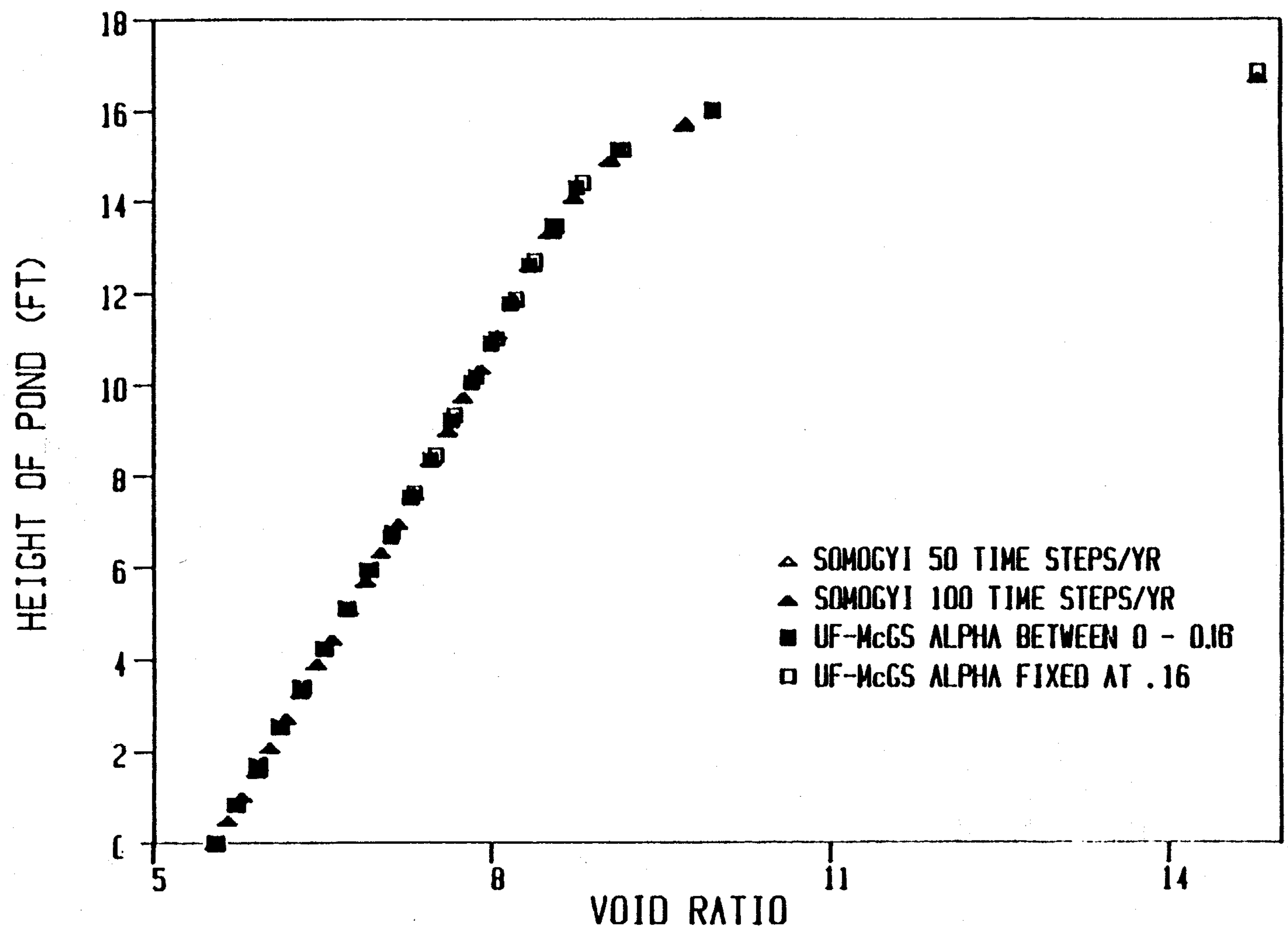

Figure 6.9. Void Ratio Profile after 3 Years for Pond 1b.

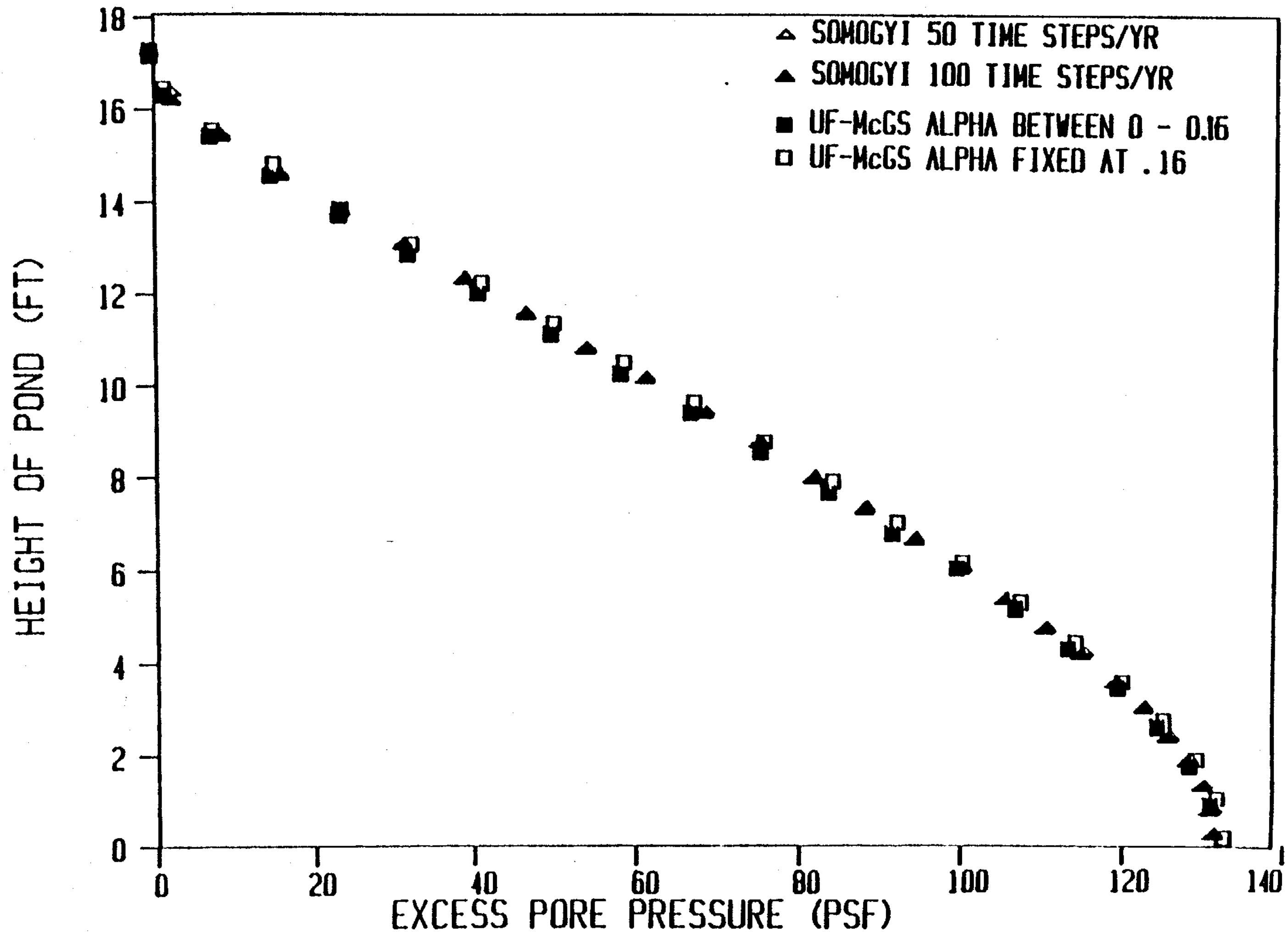

Figure 6.10. Pore Pressure Distribution after 3 Years for Pond 1b.

and 0.16. However, when the α value is fixed at .16, the UF-McGS model tends to slightly underpredict the progress of consolidation in the first two years. This may be related to the large size of the initial time steps. Altering the size of the time step for the Somogyi model from 50 to 100 time steps/yr did not provoke any divergence in results.

Figure 6.11 shows the final void ratio distribution for pond lb, while figure 6.12 compares the average degree of consolidation for the Somogyi and UF-McGS models. Results indicate that the UF-McGS model with α fixed at .16 overpredicts the consolidation settlement during the first year.

Table 6.1 shows a comparison of the final height predictions by UF-McGS, Somogyi, and the hand solution, and indicates excellent agreement.

Quiescent Consolidation with Surcharge

Pond 2a. Figure 6.13 shows a 23.6 ft homogeneous clay pond with an initial void ratio of 14.8 (S = 16%) and a 200 psf surchage. A parametric study was performed on this pond to determine the susceptibility of the UF-MCGS and Somogyi models of the number of layers dividing the pond height.

Figure 6.14 presents the progress of consolidation with time,and indicates that excellent agreement can be obtained between the UF-MCGS and Somogyi models provided a sufficient number of layers are specified. It was found that the

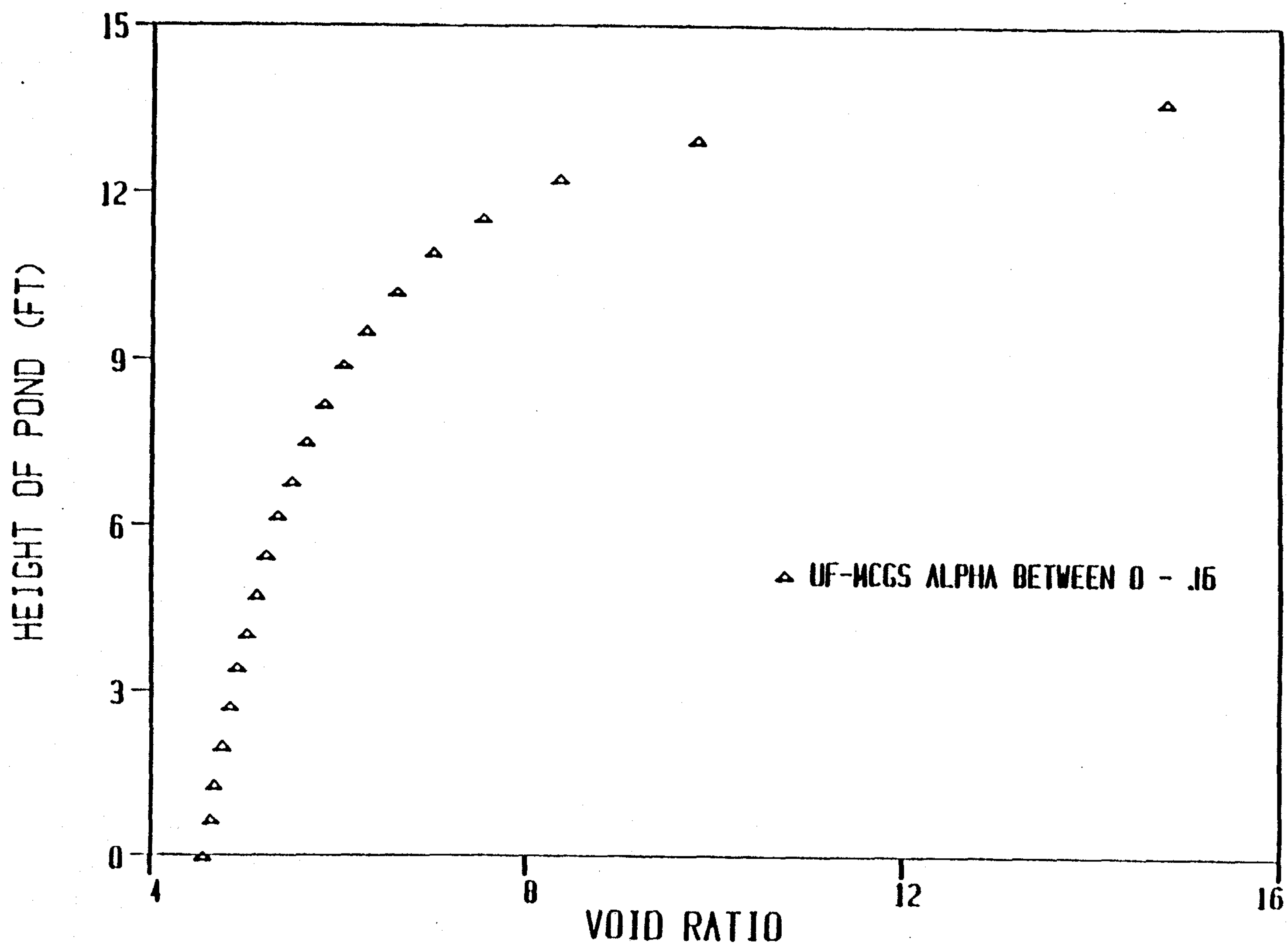

Figure 6.11. Final Void Ratio Distribution for Pond 1b.

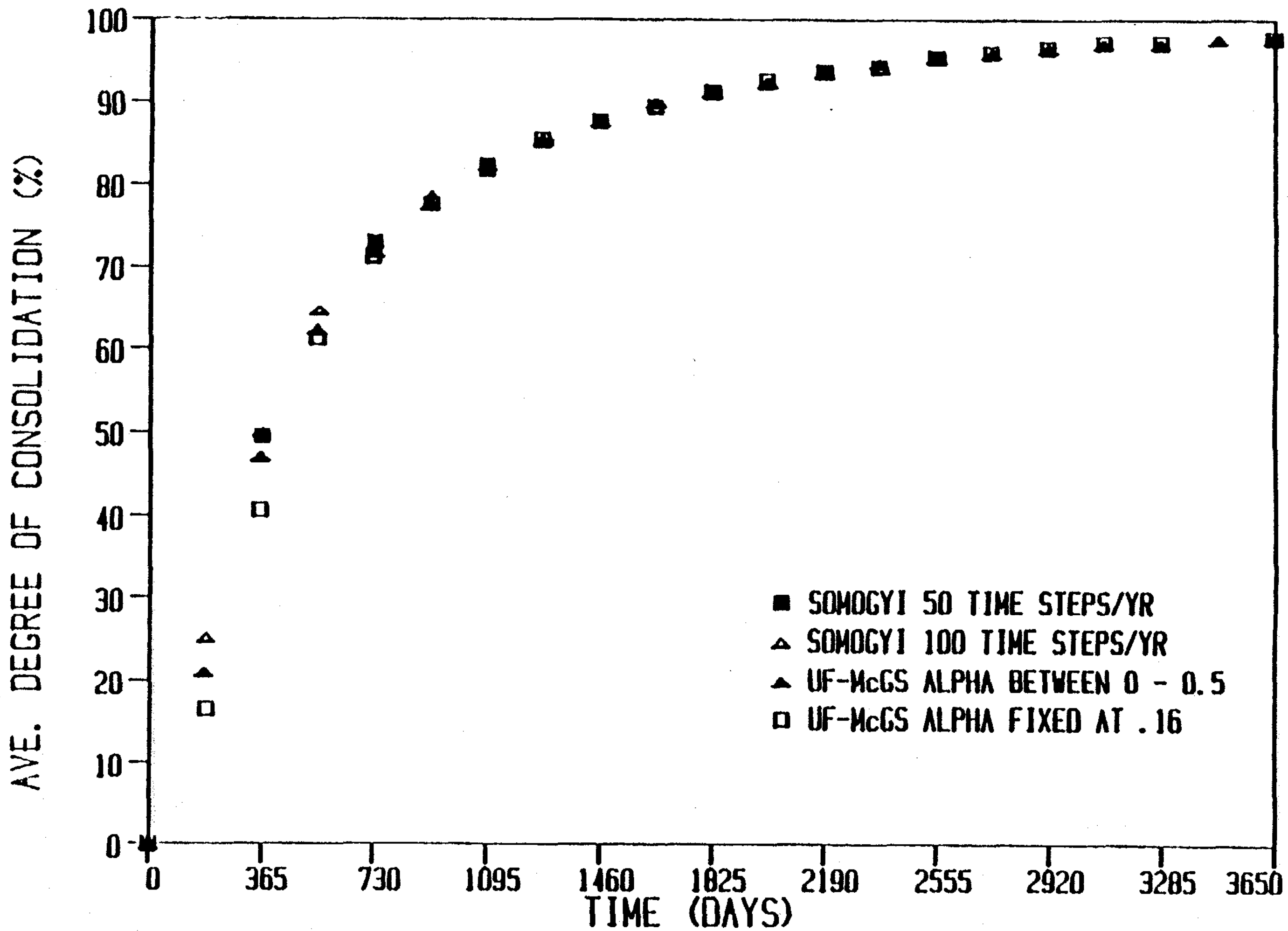

Figure 6.12. Average Degree of Consolidation vs. Time for Pond 1b.

Table 6.1. Summary of Results for Quiescent Consolidation

	Final Height (ft)	Time (yrs)
<u>Pond 1a</u>		
UF-McGS	7.80	11.5
Somogyi	7.75	-
Closed Solution	7.73	-
<u>Pond 1b</u>		
UF-McGS	13.65	33.97
Somogyi	13.56	-
Closed Solution	13.53	-

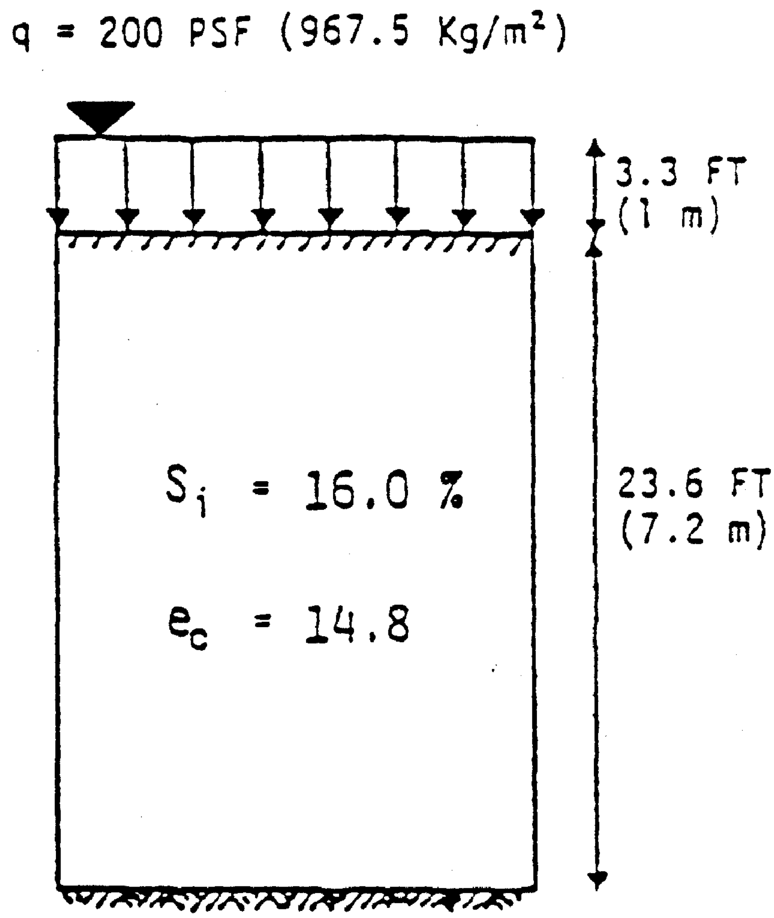

Figure 6.13. Pond 2a: Quiescent Consolidation with Unifor: Void Ratio and Surcharge.

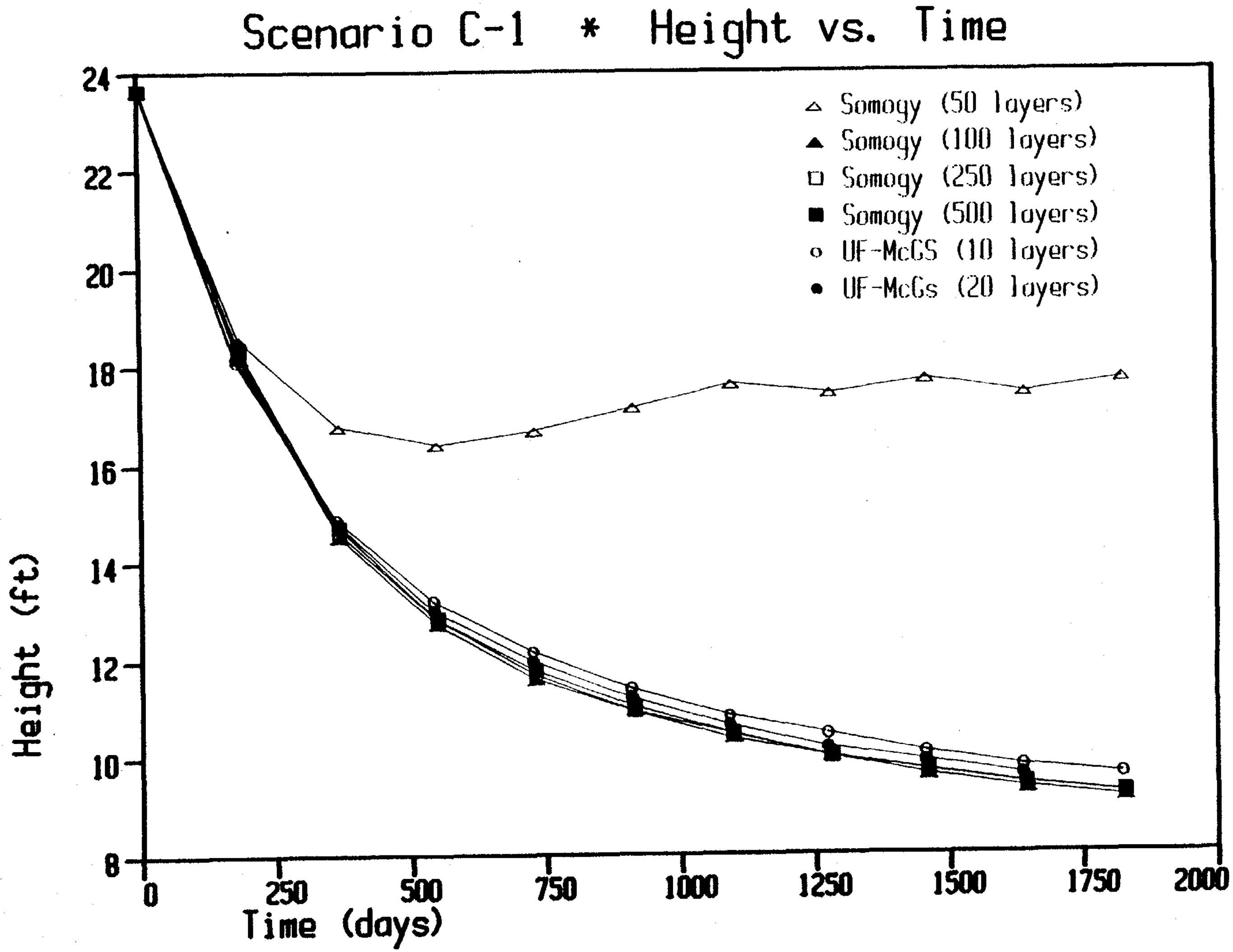

Figure 6.14. Height versus Time for Pond 2a.

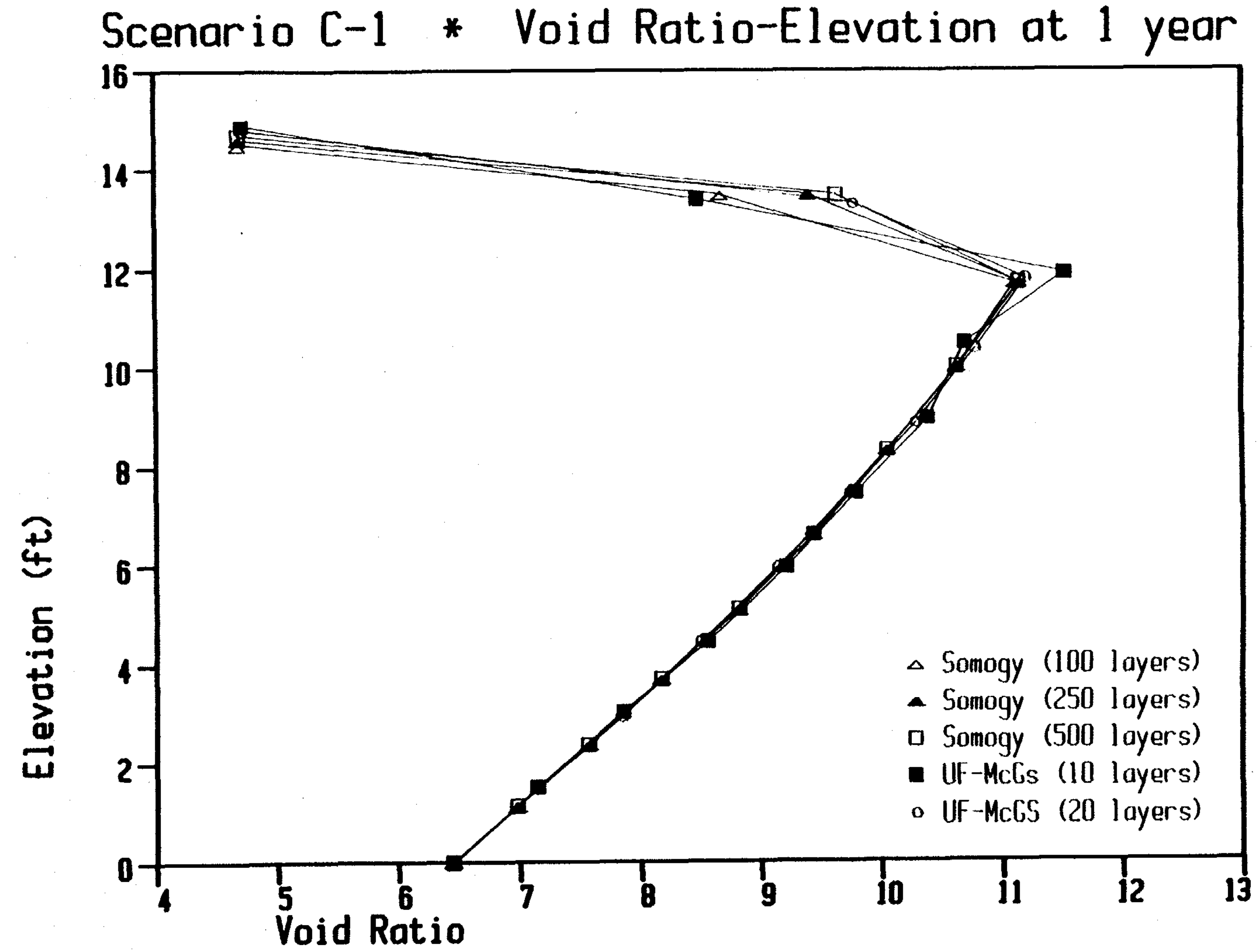

Figure 6.15. Void Ratio Profile for Pond 2a.

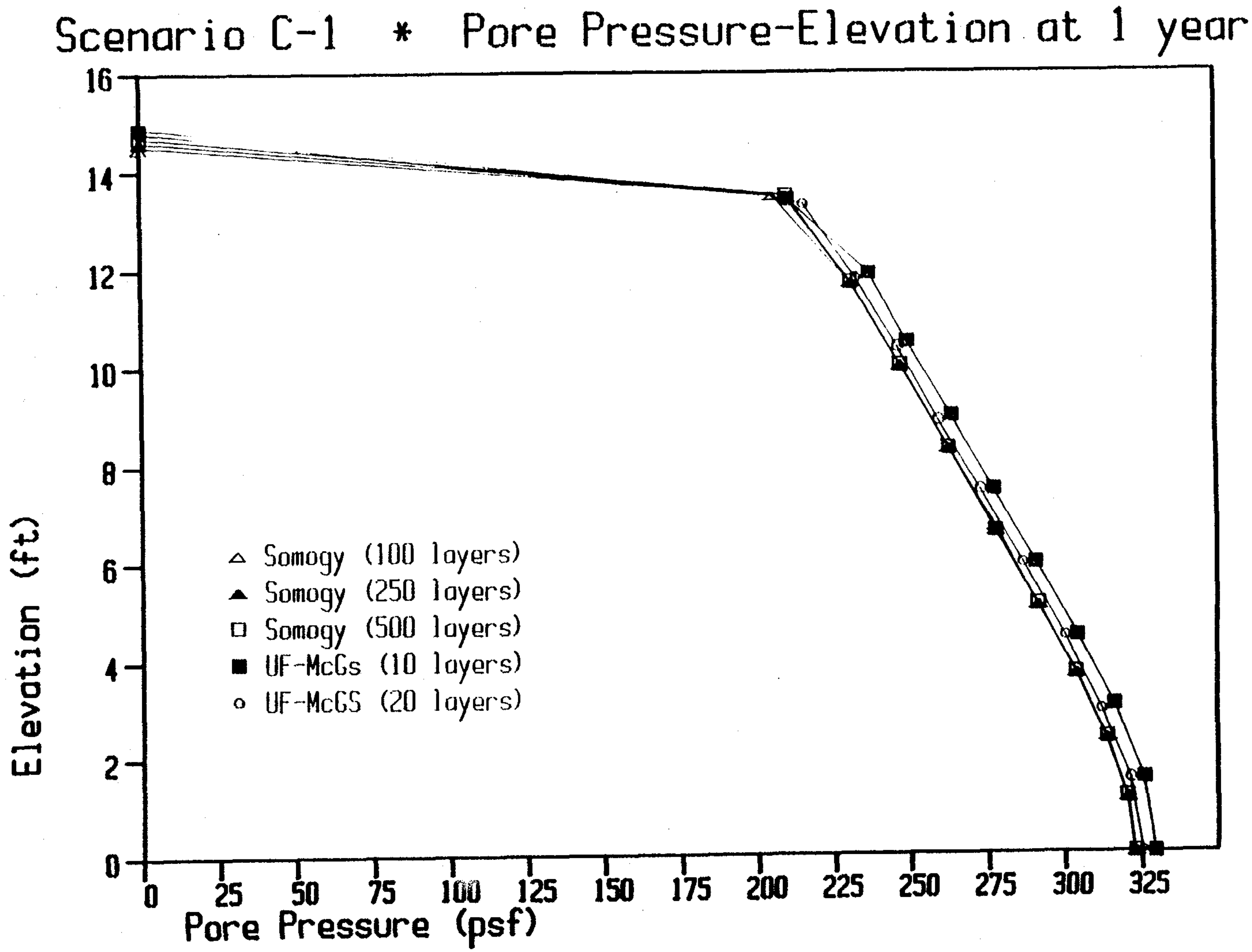

Figure 6.16. Pore Pressure Profile for Pond 2a.

Table 6.2. Summary of Results for Surcharged Pond 2b.

	Final Height (ft)	Time (yrs)
Pond 2b		
UF-McGS	8.11	31.95
Hand Solution	8.01	-

Somogyi model results are quite dependent upon the number of layers used, and over 100 layers were required for agreement. [We also used 1000 time steps, following the recommended 10 time steps/year]. Table 6.2 summarizes the final pond heights as predicted by the UF-MCGS, Somogyi, and closed form solution.

Figures 6.15 and 6.16 respectively present the void ratio and pore pressure profiles after one year for the UF-MCGS and Somogyi models. As shown in figure 6.15, the UF-MCGS with 10 layers slightly disagrees with other values in the top regions. Similarly the pore pressure results presented in Figure 6.16 show the 10 layer UF-McGS definition produces a higher pore pressure profile. The presence of the surcharge at a surface drainage boundary causes both models to predict a sharp change in both void ratio and pore pressure just beneath the pond su rface. This change may merely be an artifact of these models and boundary conditions.

<u>Pond 2b.</u> Figure 6.17 shows a 23.6 pond with variable void ratio and a 200 psf surcharge. The results obtained from the UF-McGS model cannot be compared to the Somogyi model due to its inability to analyze non-homogeneous layers. However, the final height predictions by UF-McGS and the closed form solution are presented in Table 6.2 and show excellent agreement.

Figure 6.18 compares the progress of consolidation for

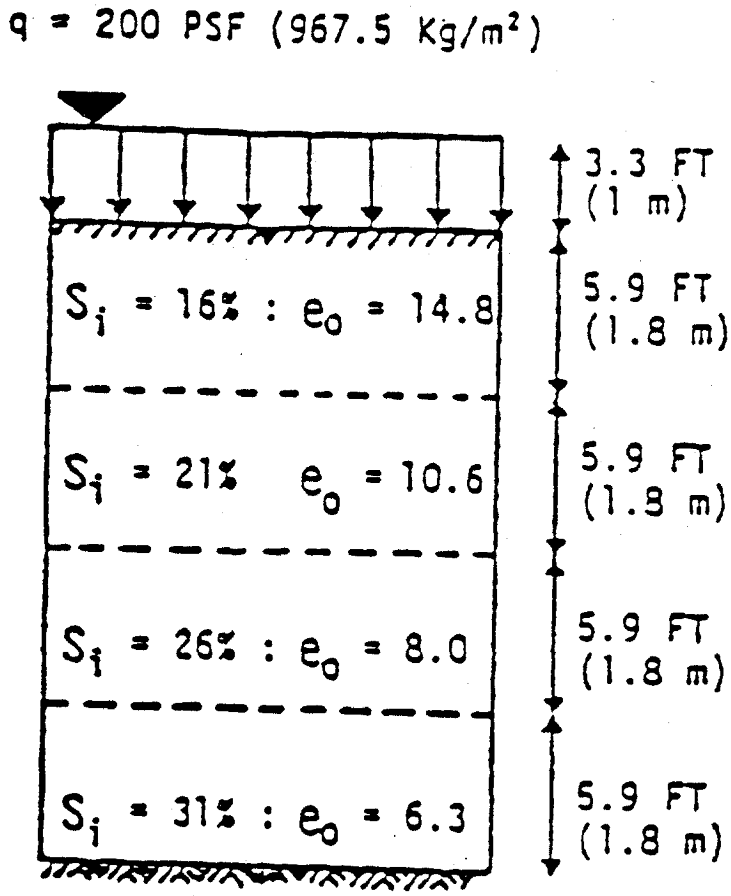

Figure 6.17. Pond 2b: Quiescent Consolidation with Variable Void Ratio and Surcharge.

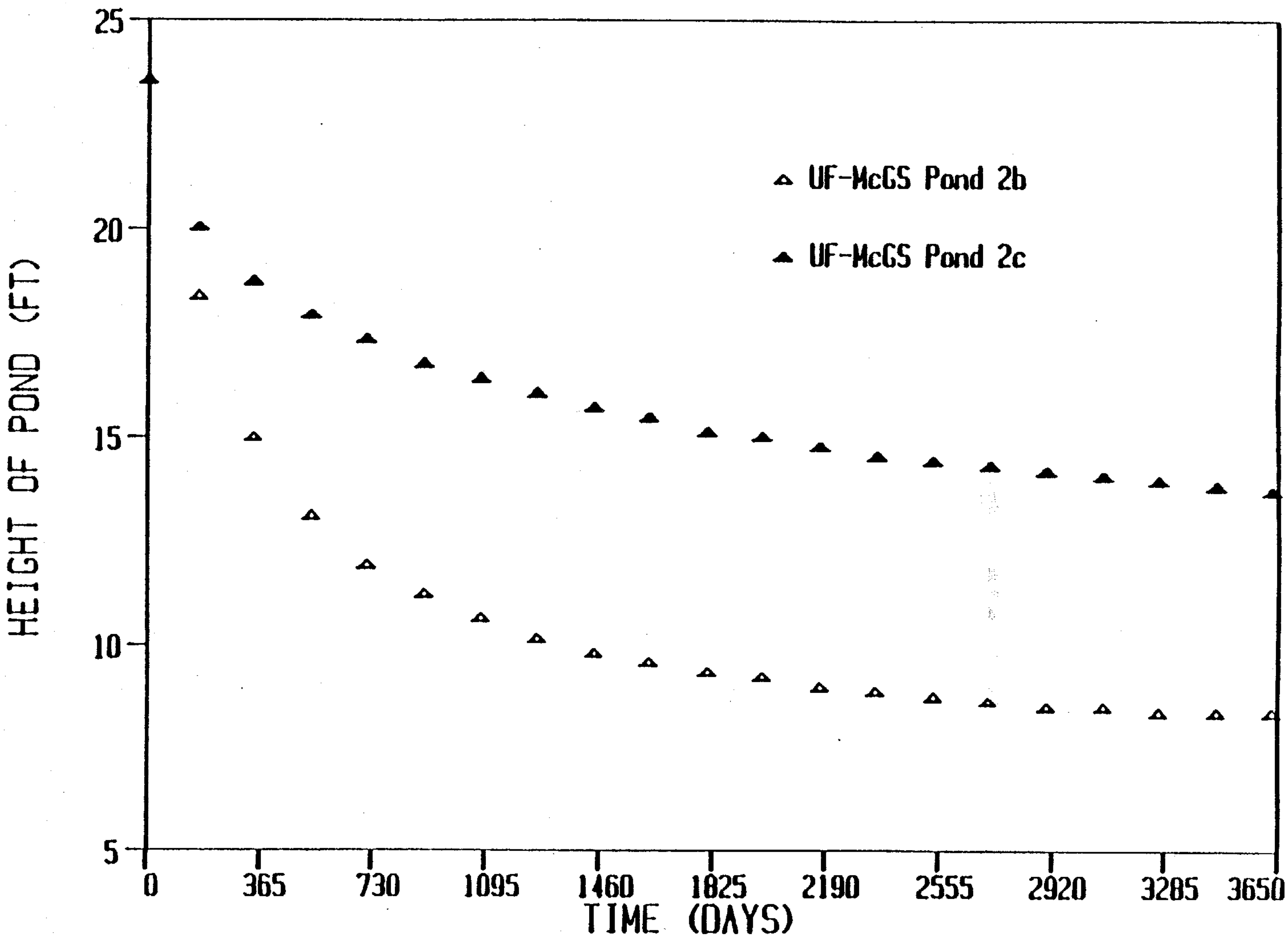

Figure 6.18. Comparison of Height vs. Time for Ponds 2a and 2b.

ponds 2a and 2b and elucidates the fact that pond 2a settles 4.4 ft. more than pond 2b. This is because pond 2a has a lower initial solids content.

Figure 6.19 compares the final void ratio profiles for ponds 2a and 2b and indicates that pond 2b has a lower final void ratio than pond 2a because the former has a higher initial solids content.

Figure 6.20, which gives the progress of the average degree of consolidation with time for ponds 2a and 2b, shows that the rate of consolidation of pond 2b is slower than pond 2a.

Figures 6.21 and 6.22 present the void ratio and pore pressure profiles at one year.

Stage Filling

Pond 3a. Figure 6.23 (1) shows a pond filled in two 6 month increments separated by a 6 month quiescent increment with clay at a void ratio of 22.82, with a filling rate of .0656 ft/day. This pond simulates waste clay ponds which are filled intermittently with thickened clays pumped from an initial settling area.

In order to obtain a prediction for the Somogyi model, the program FLINS was utilized to model the inital filling, then its output became the input to QSNS and finally FLCOS was used to model later filling.

Figure 6.24 compares the progress of consolidation for the Somogyi model and the UF-McGS model. Results indicate

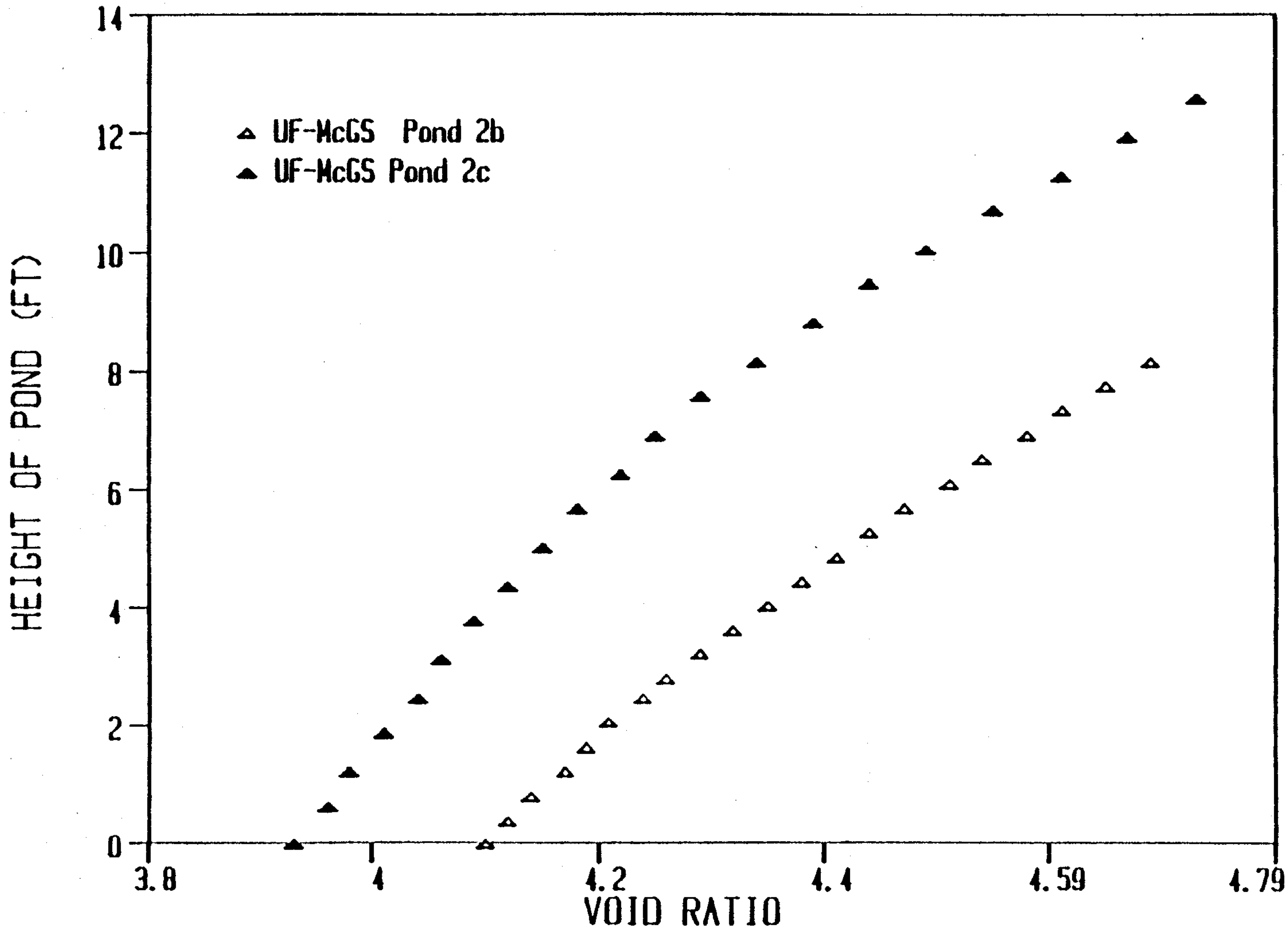

Figure 6.19. Final Void Ratio Distribution for Ponds 2a and 2b.

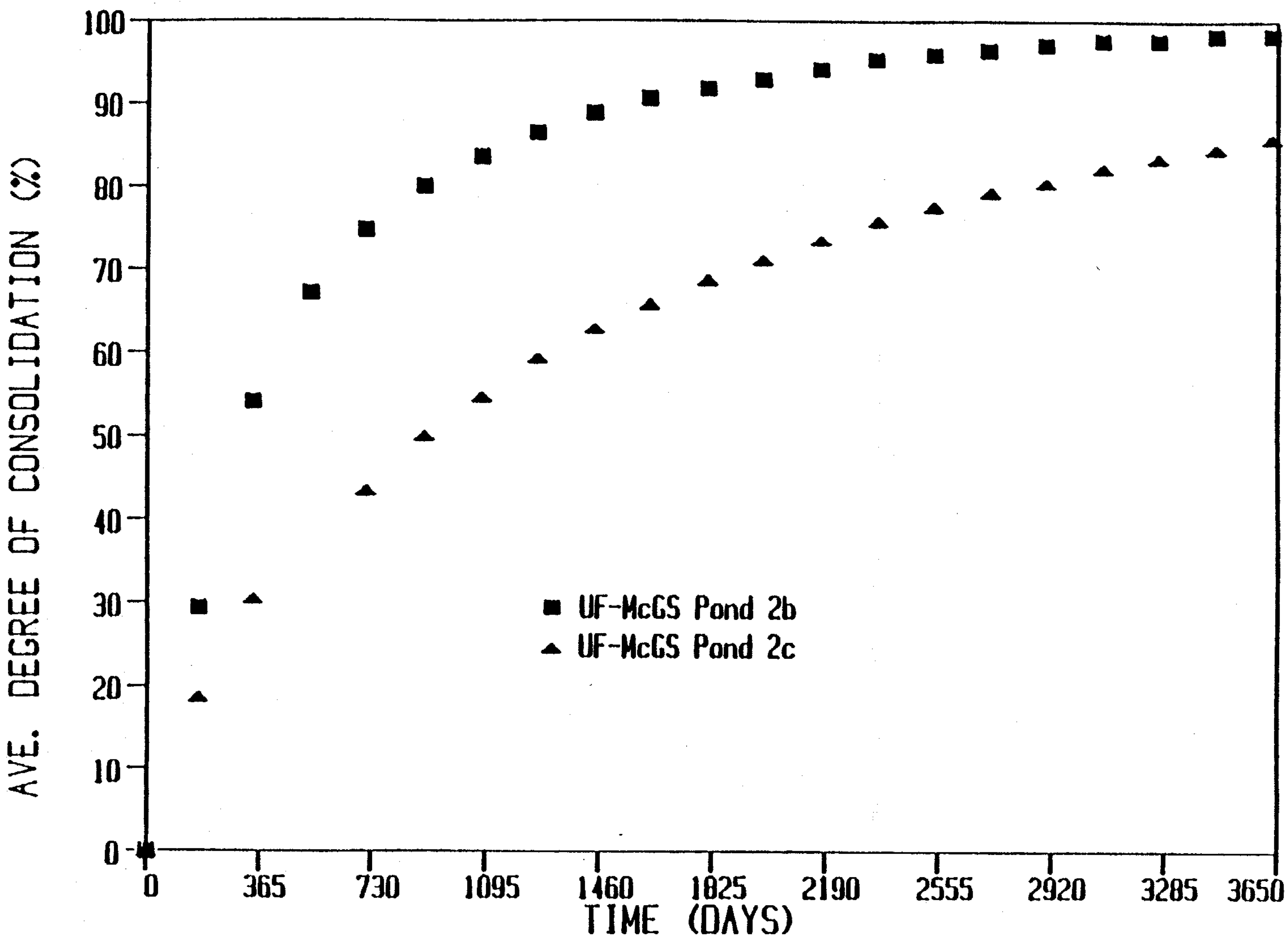

Figure 6.20. Average Degree of Consolidation for Ponds 2a and 2b.

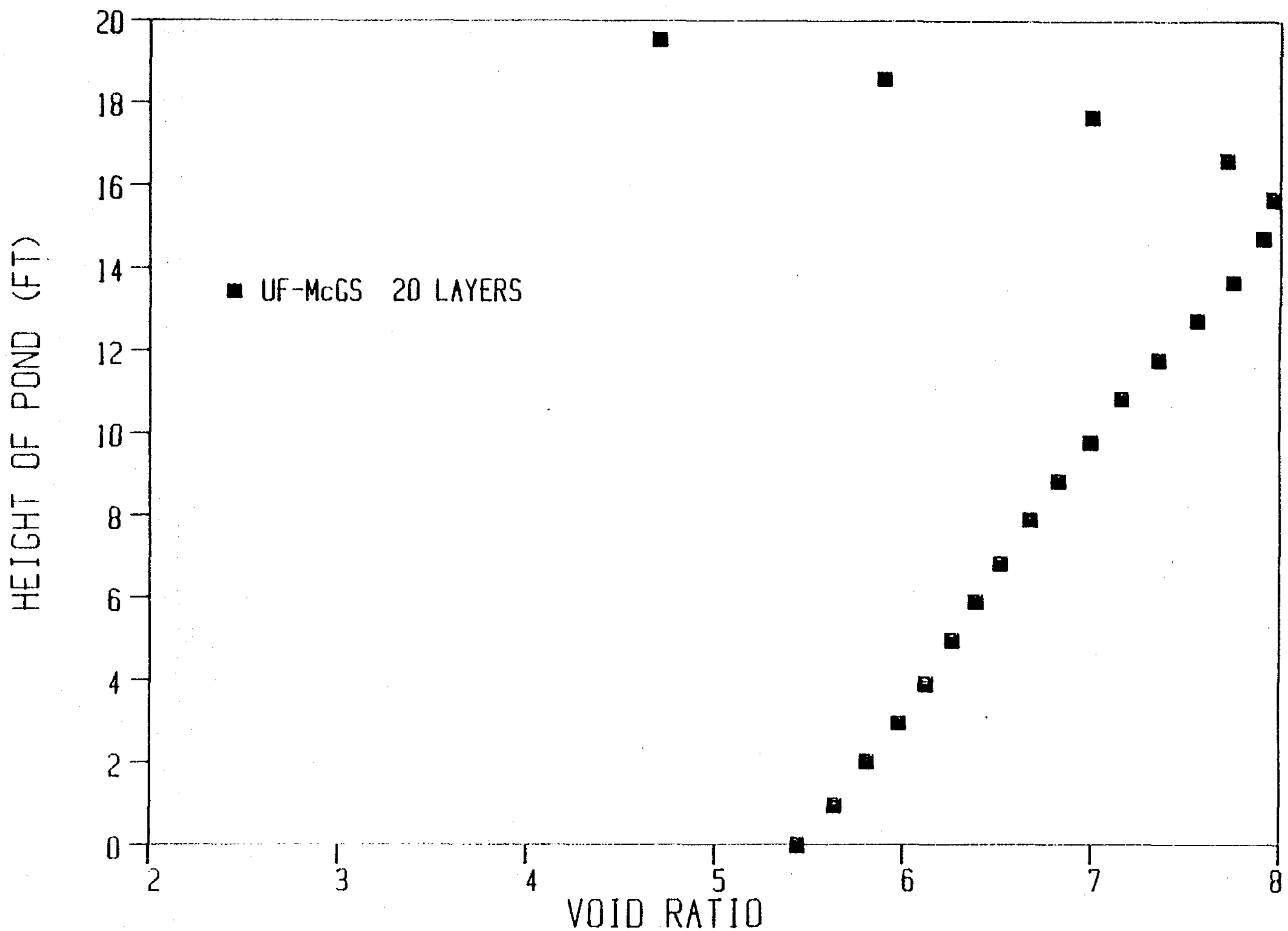

Figure 6.21. Void Ratio Profile at 1 year for Pond 2b.

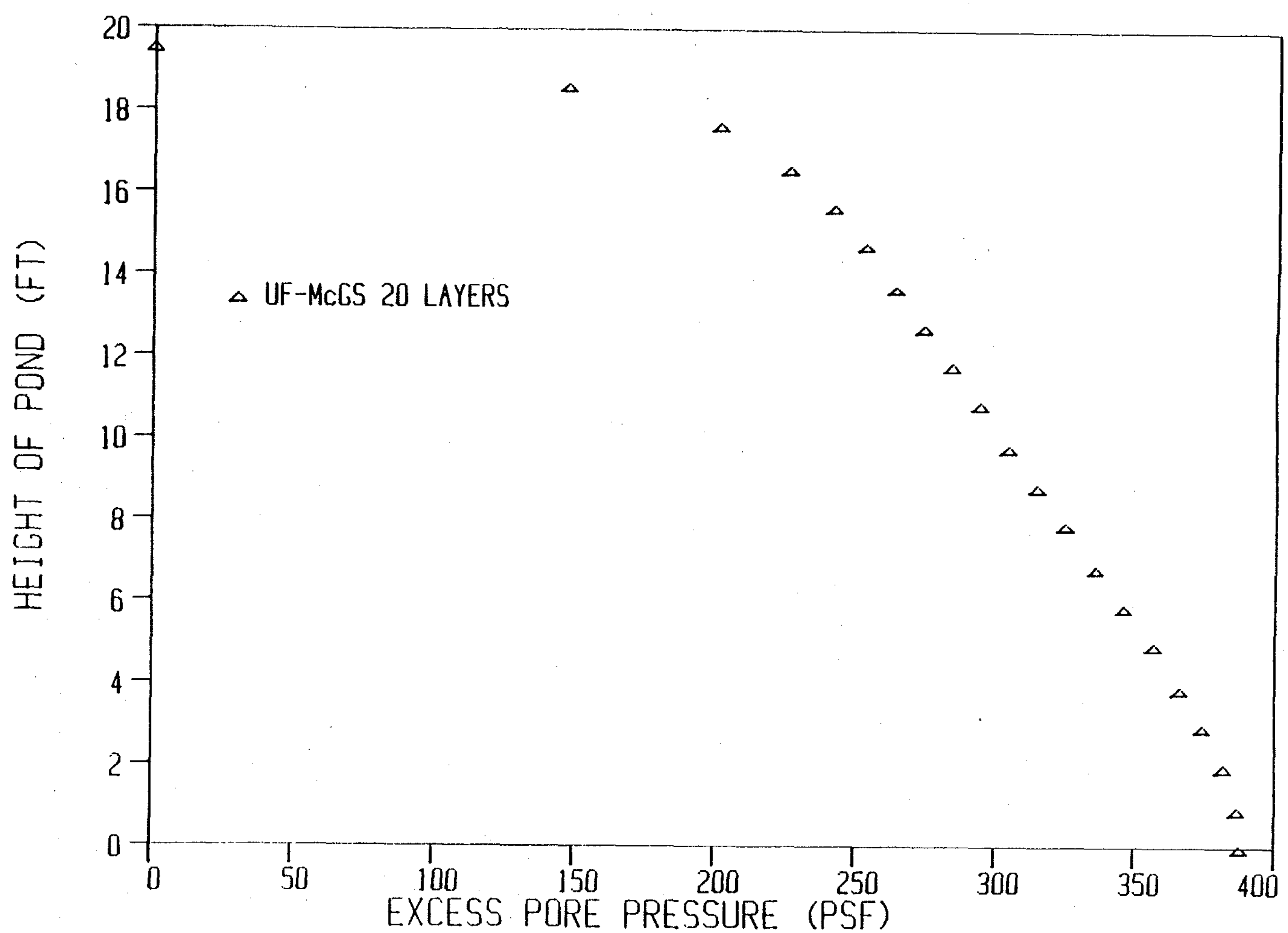

Figure 6.22. Excess Pore Pressure Profile at 1 year for Pond 2b.

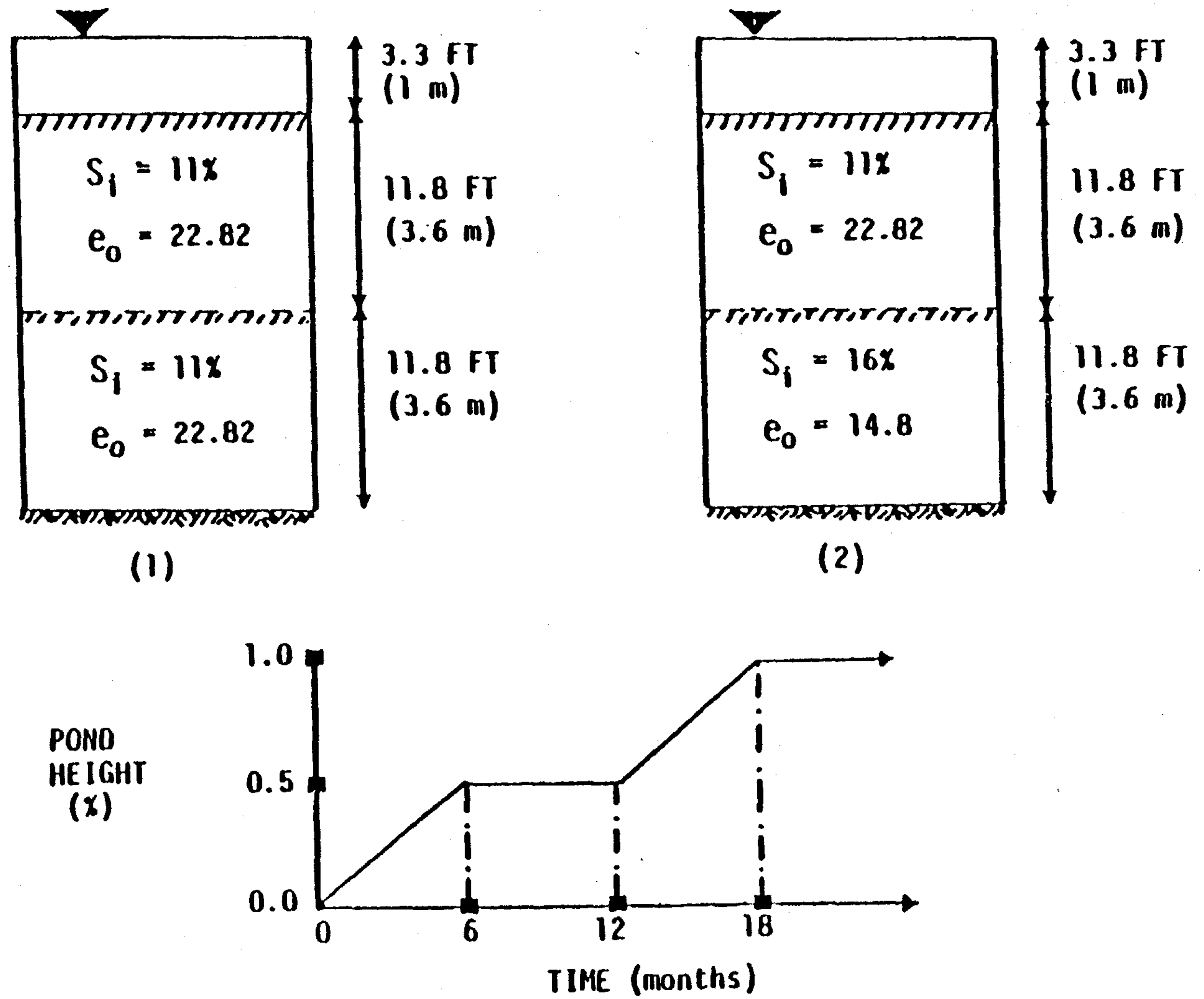

Figure 6.23. Stage Filling: (1) Pond 3a. Uniform Void Ratio.
(2) Pond 3b. Non-Uniform Void Ratio.

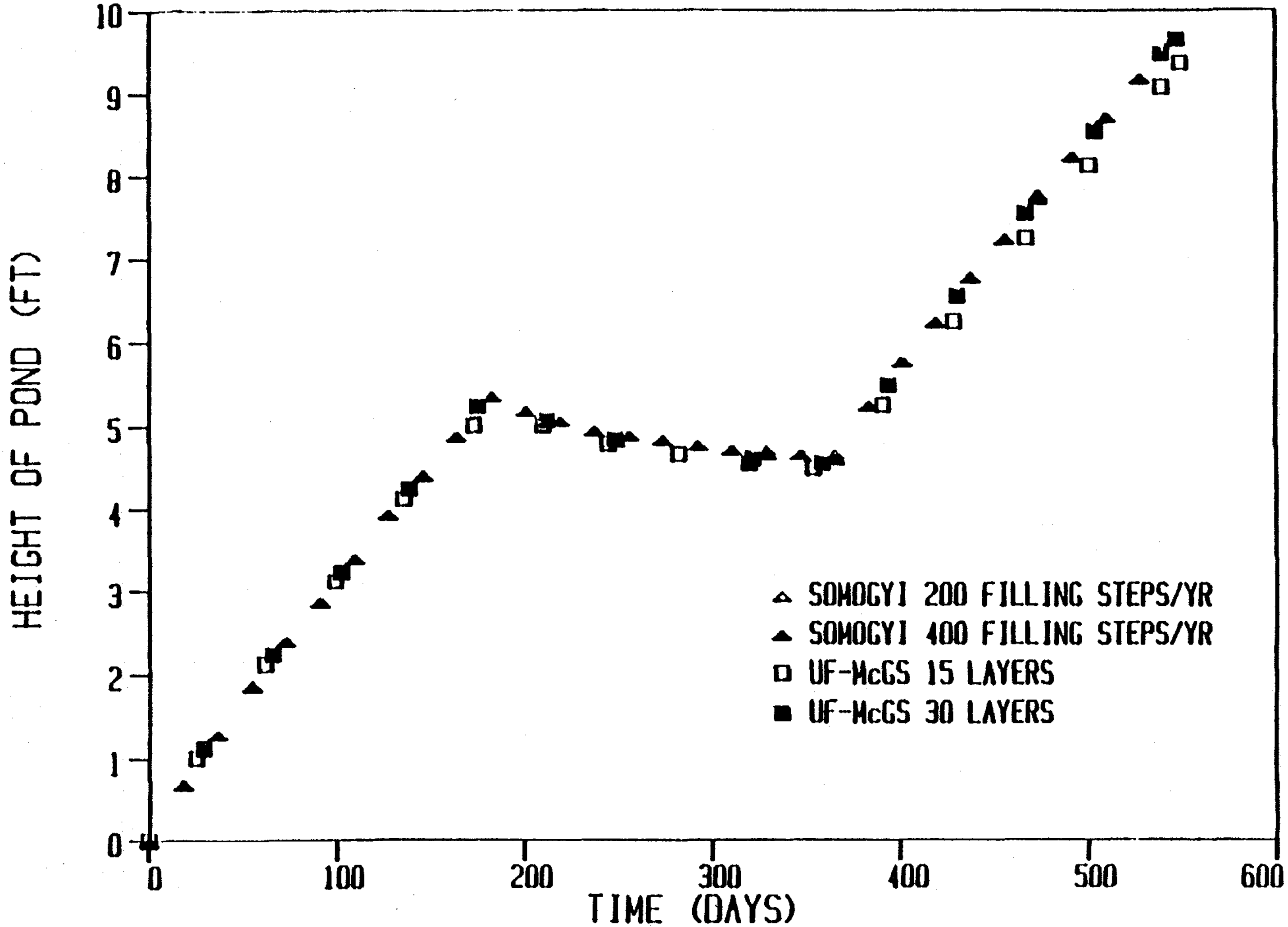

Figure 6.24. Height vs. Time for Pond 3a.

very close agreement between both Somogyi runs and UF-MCGS at 30 layers. However, the 15 layer UF-McGS analysis shows slight disagreement with the other runs towards the end of the second filling period. This is probably due to an interpolation error since layer thickness after 500 days is substantial.

Figure 6.25 depicts the variation in the average degree of consolidation with time. It is interesting to note from this graph that during the periods of quiescent consolidation, the values increased, while during periods of filling, the average degree of consolidation tended to decrease. Also, it should be noted that during the first period of quiescent settling, the deposit almost became fully consolidated.

Figure 6.26 compares the 1 year void ratio profile for pond 3a and shows excellent agreement.

<u>Pond 3b.</u> Figure 6.23 (2) shows a pond filled in two 6 month increments separated by a 6 month quiescent increment with clay at a void ratio of 14.8 during the first increment and at a void ratio of 22.82 during the second increment, with a filling rate of .0656 ft/day.

The Somogyi analysis for this pond was performed in similar fashion as pond 3a. Figure 6.27 displays the progress of consolidation with time. This figure shows excellent agreement between all three runs during continuous fill and a slight disagreement between the Somogyi runs and the UF-McGS run during quiescent consolidation. This

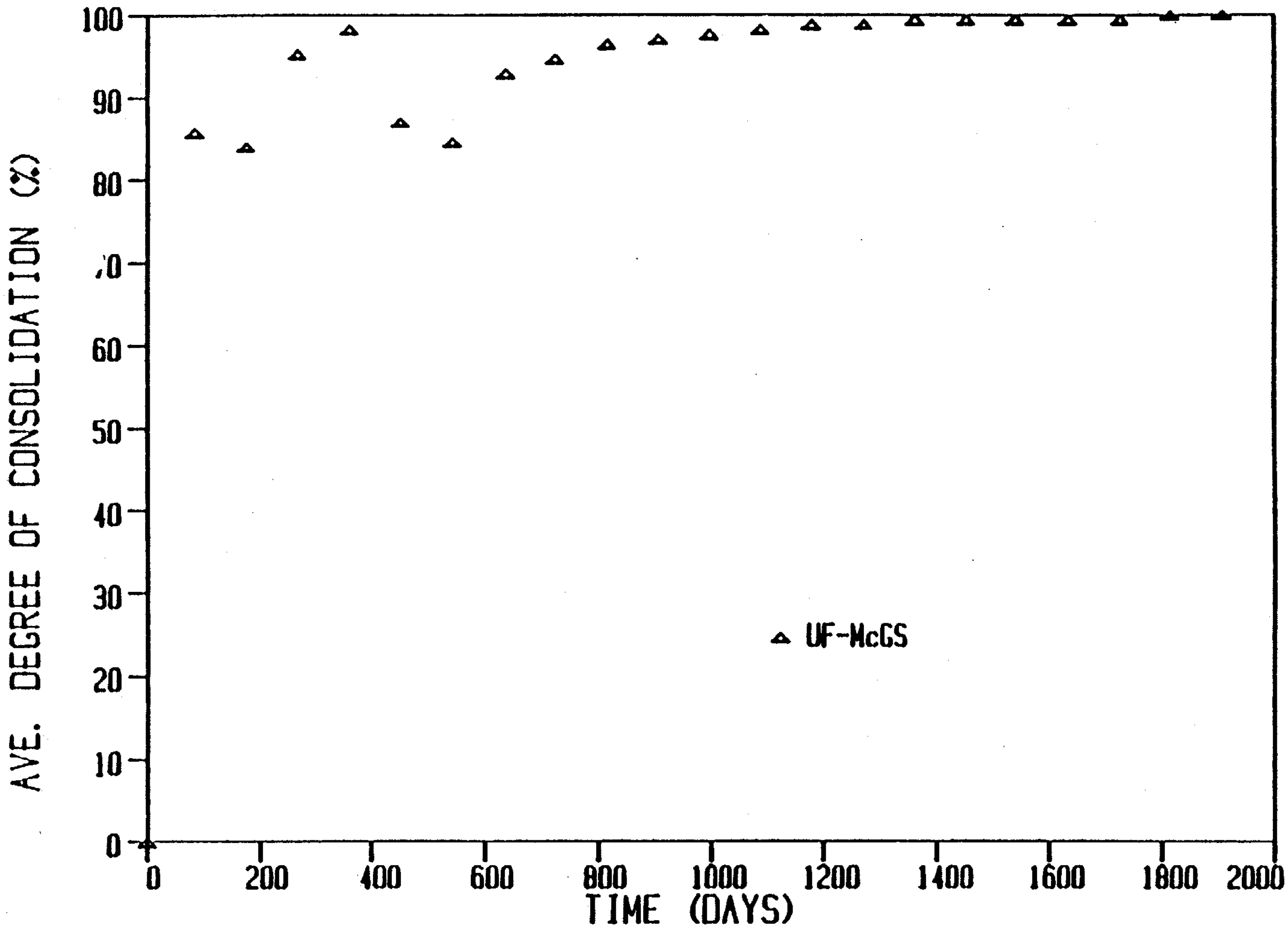

Figure 6.25. Average Degree of Consolidation vs. Time for Pond 3a.

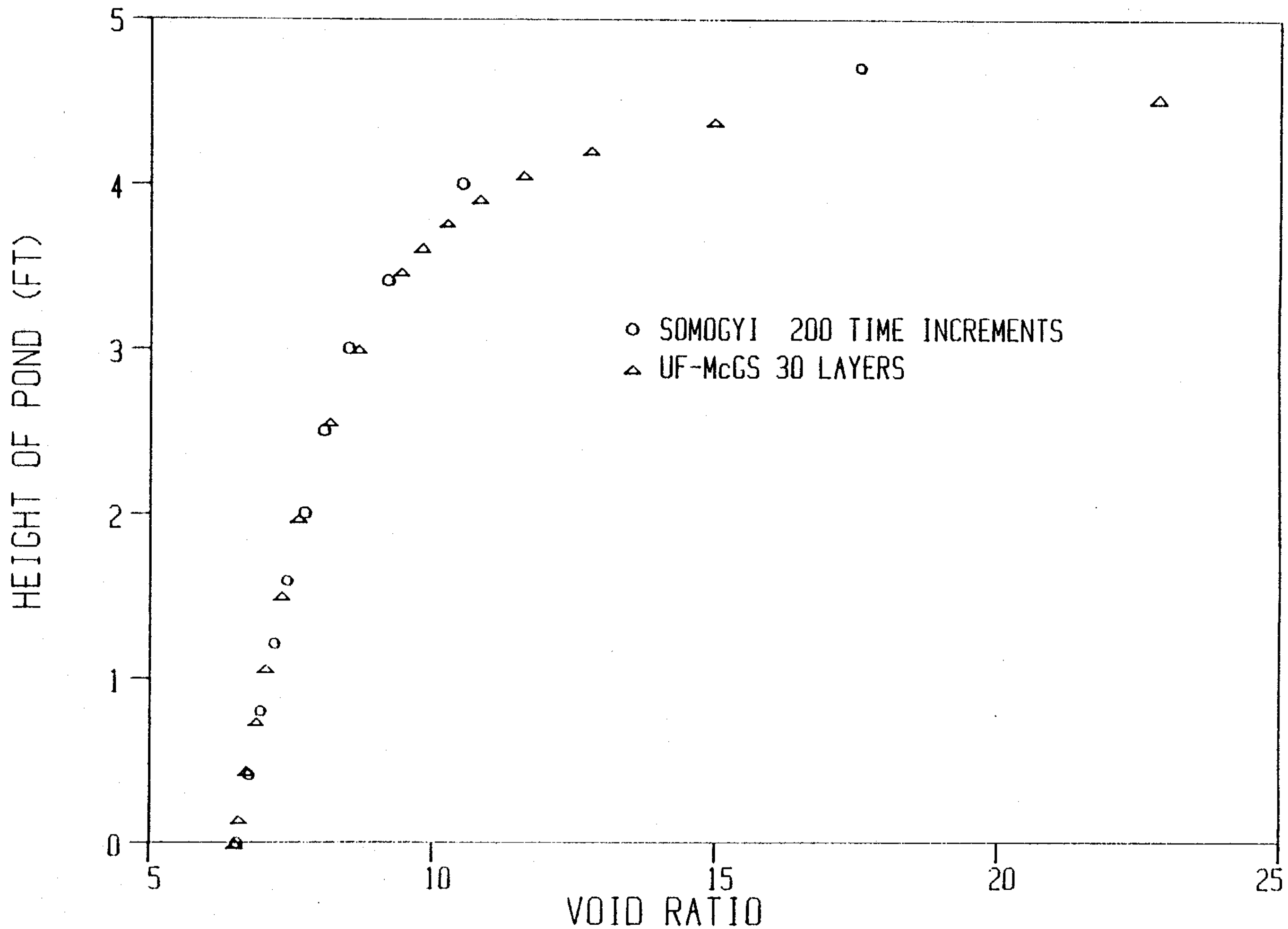

Figure 6.26. One year Void Patio Profile for Pond 3a.

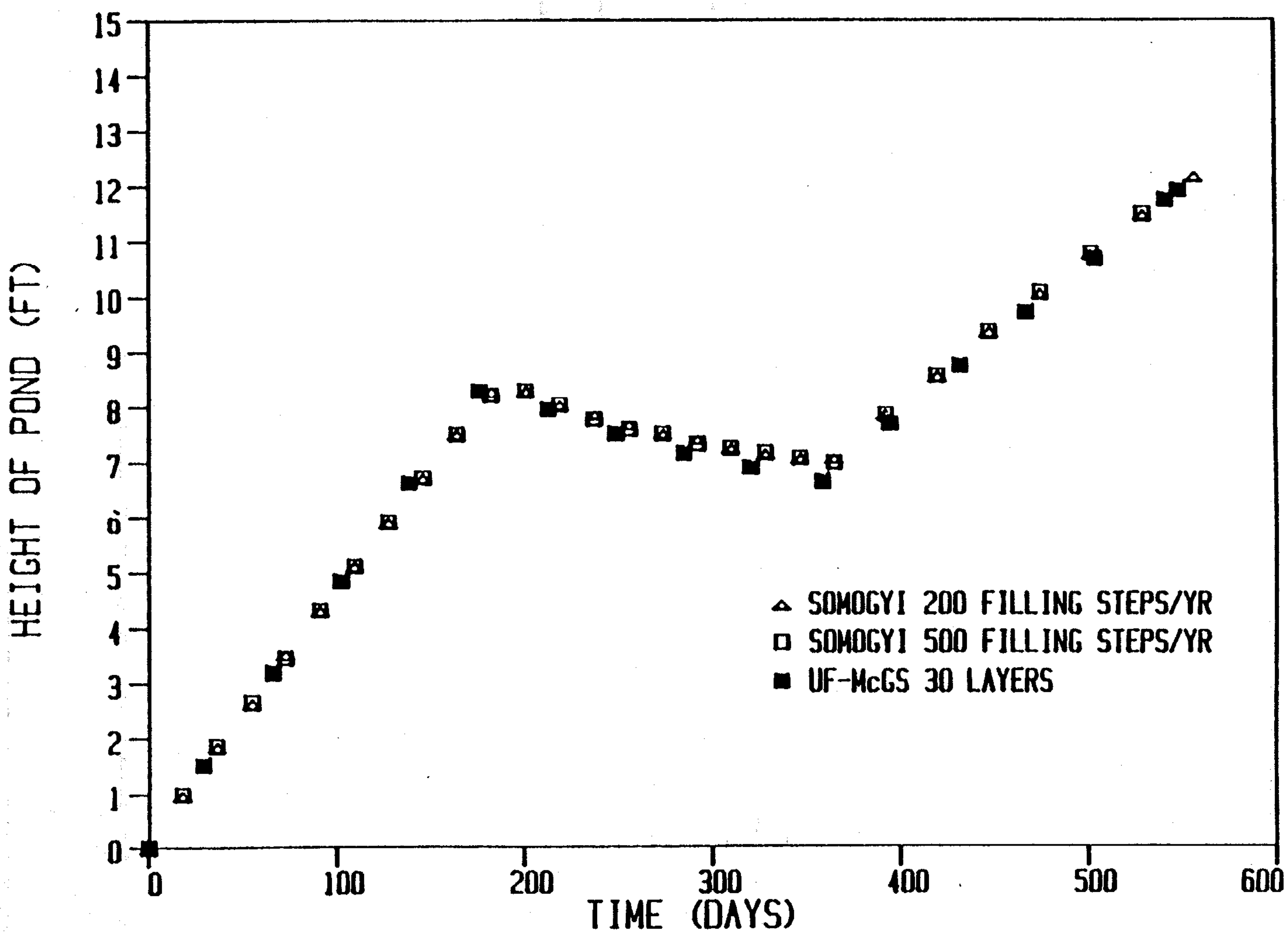

Figure 6.27. Height vs. Time for Pond 3b.

difference may be pinpointed to the fact that when the FLINS output becomes the QSNS input, the new "starting height of pond" is recalculated based on the layer thickness inputted by the user as obtained from FLINS. For the 200 filling steps/year case, the final height given by FLINS disagreed with the starting height of QSNS by .6 ft. Furthermore, pond 3b has a void ratio discontinuity because the solids content for the top layer is different from that for the lower layer. The Somogyi model assumes that the solids content at the surface of an existing deposit is identical to that attained by the new material after deposition (Somogyi et al.,1981), while the UF-McGS model simply approximates the boundary condition by interpolating the boundary solids content. Thus,the Somogyi model has stability problems for this analysis (500 filling steps/yr did not converge).

Figure 6.28 shows a plot of the average degree of consolidation vs. time. This figure shows the same trends as figure 6.25. However, since the solids content for pond 3b is higher than for pond 3a, the progress of consolidation is slower and a pronounced local peak is observed during filling at 11 % solids content.

Figure 6.29 presents the void ratio profile of pond 3b after one year for the Somogyi and UF-MCGS models. There results show an excellent agreement.

Two Layer Quiescent Consolidation

Ponds 4a and 4b. Figure 6.30 elucidates ponds similar

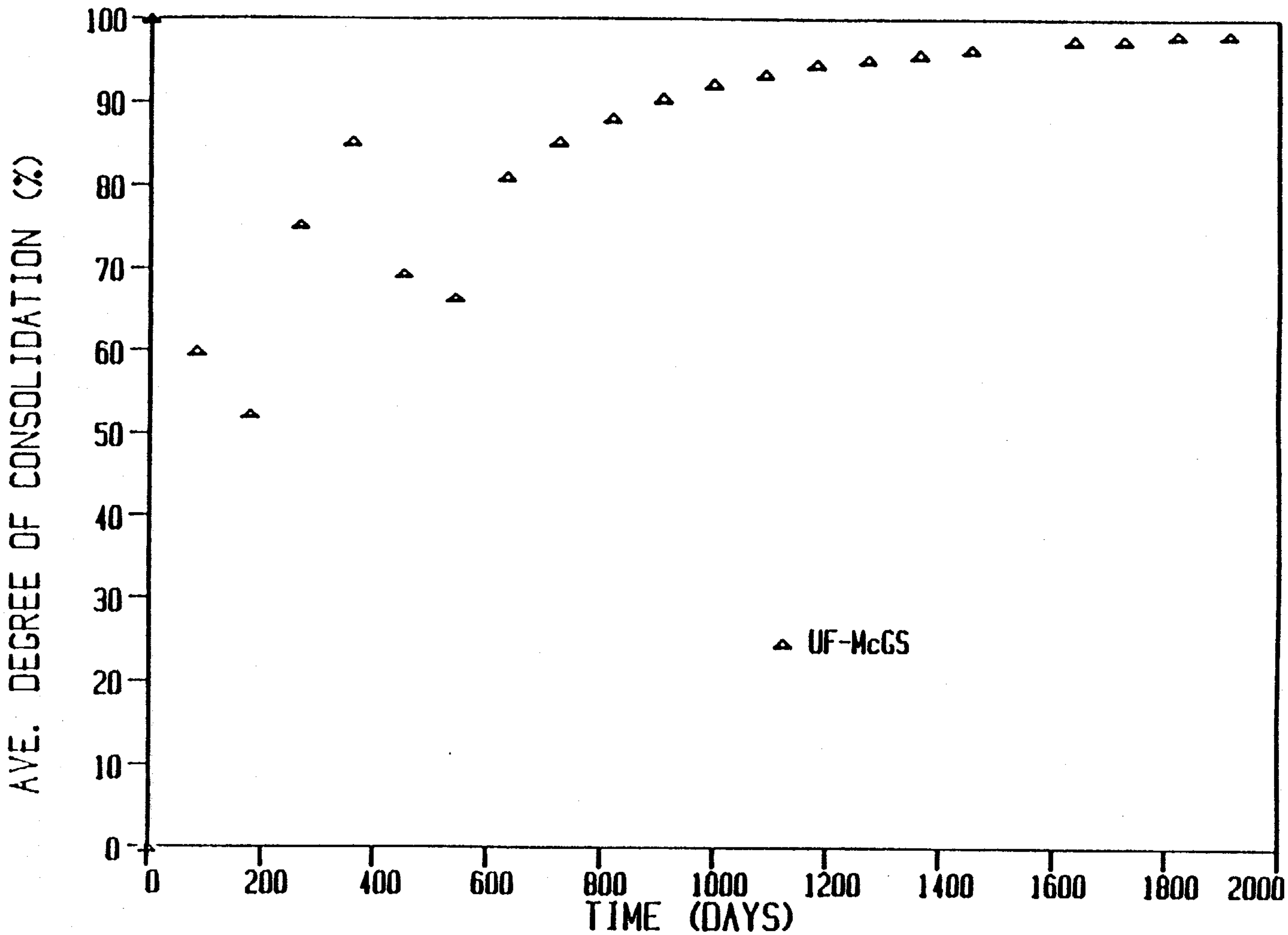

Figure 6.28. Average Degree of Consolidation vs. Time for Pond 3b.

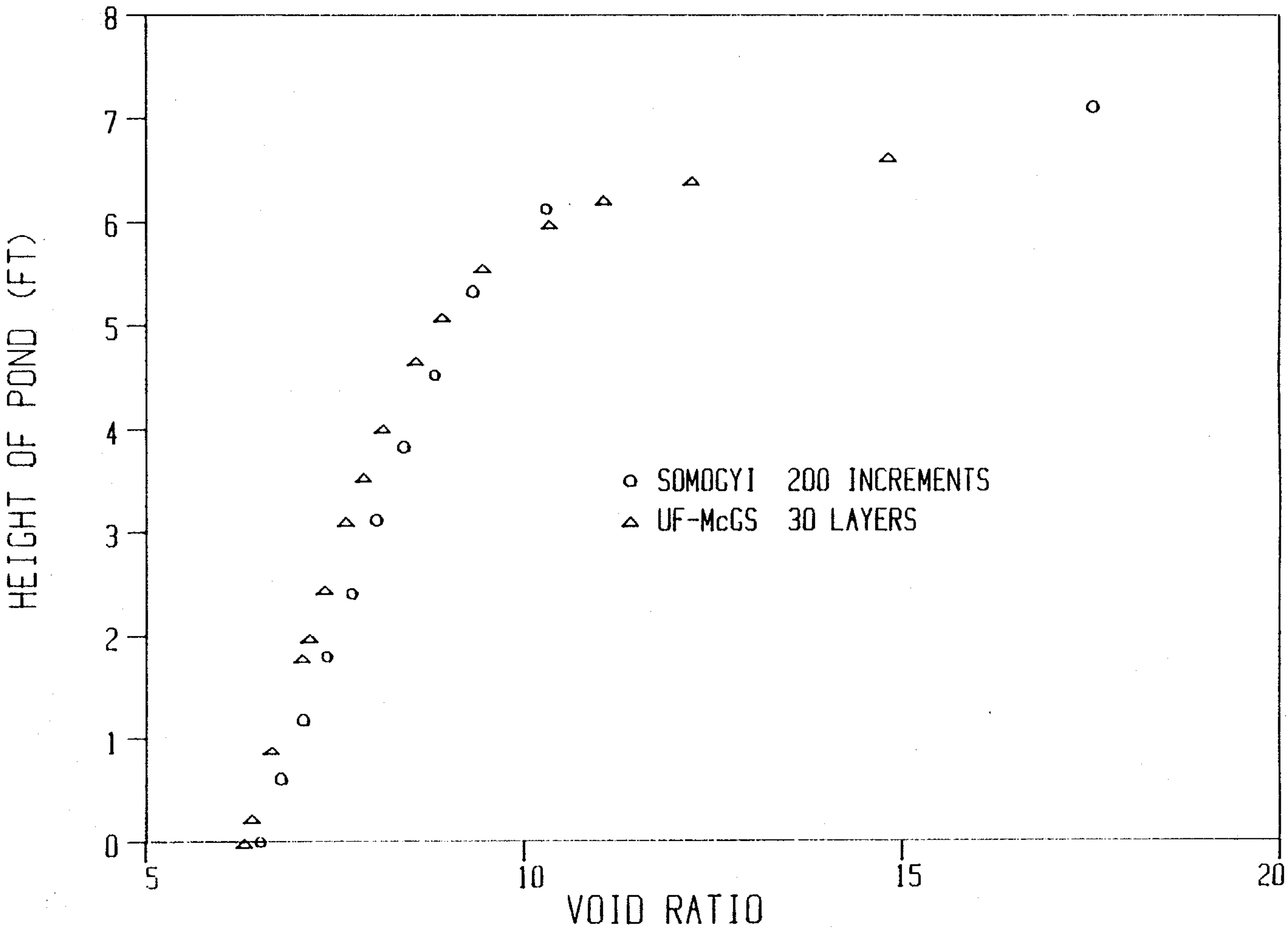

Figure 6.29. One year Void Ratio Profile for Pond 3b.

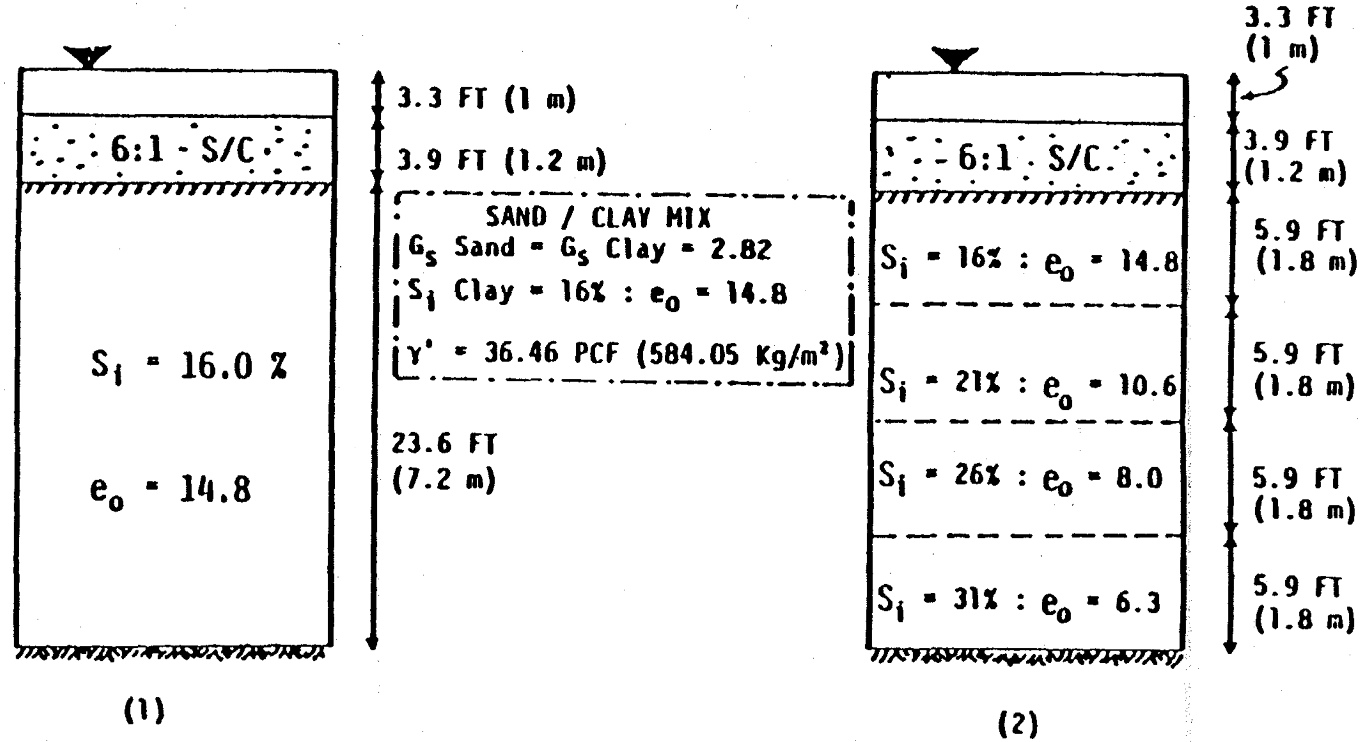

Figure 6.30. Two Layer Quiescent Consolidation: (1) Pond 4a. Uniform Void Ratio
(Sand/Clay Cap Surcharge) (2) Pond 4b. Variable Void Ratio

to 2b, and 2c; however instead of a sand surcharge, these ponds are subjected to a surcharge load by a 6:1 sand/clay cap with different properties from the underlying waste clay. Because of the different layers of material, it is impossible to obtain the time of consolidation using the UF-McGS model; however,the height of the pond may be obtained by analyzing the bottom layer, considering the top as surcharge, and then considering the top layer as consolidating by itself, since properties are available, and adding the change in height for each layer.

Table 6.3 presents the final heights obtained for the UF-McGS and Somogyi model for both ponds 4a and 4b, as well as the closed form solution. There comparisons show excellent agreement between all three methods.

Table 6.3. Summary of Results for Symposium Ponds with Sand /Clay Cap

	Final Height (ft)
<u>6/1 Sand Clay Cap</u>	
UF-McGS	3.26
Somogyi	3.25
Closed Form Solution	3.20
<u>Pond 4a</u>	
<u>Waste clay with Surcharge</u>	
UF-McGS	8.53
Final Height = Sand Clay cap + waste clay = 11.78 ft.	
Somogyi	8.40
Final Height = 3.25 + 8.40 = 11.65 ft.	
Closed Form Solution	8.35
Final Height = 3.20 + 8.35 = 11.55	
<u>Pond 4b</u>	
<u>Waste clay with Surcharge</u>	
UF-McGS	13.00
Final Height = Sand Clay cap + waste clay = 16.25 ft.	
Closed Form Solution	12.58
Final Height = 3.20 + 12.53 = 15.78 ft.	

CHAPTER SEVEN
CONCLUSIONS AND RECOMMENDATIONS

Review of Objectives

The major objectives of this study were as follows:

1. To modify an existing piecewise linear computer program.
2. To compare spatial vs. reduced representation finite strain non - linear consolidation theory.
3. To predict a series of model ponds to be discussed at the "Symposium on Consolidation and Disposal of Phosphatic and Other Waste Clays," Lakeland, 1987.
4. To develop a multiple layer piecewise linear large strain consolidation model.

Conclusions

1. All valid large strain consolidation equations can be derived from a single general equation.
2. Spatial piecewise linear theory and GEH theory are identical and only differ in the coordinate system used for solution.
3. Spacial piecewise linear programs are more versatile than GEH programs since they can model non-homogeneous deposits.
4. The UF-McGS and the Somogyi models show excellent

agreement for quiescent consolidation, quiescent consolidation with surcharge, and continuous fill with uniform solids content.

5. The Somogyi model is consistently faster than the UF-McGS model; however, the latter executes within 30 minutes of the Somogyi model, unless cut into many layers (over 30), or modelling initial filling.
6. For quiescent consolidation, the UF-McGS is most efficient and very accurate at 20 layers and 25 points defining the material parameter curves.
7. For quiescent consolidation, the Somogyi model is very efficient at 50 layers and 100 time steps/yr.
8. At values higher than 10, the number of points defining the material parameter curves for the UF-McGS have no significant bearing on the accuracy of predictions.
9. Fixing the α parameter to the rapid convergence value of .16 in the UF-McGS model may create large time steps which may not properly account for nonlinearity.
10. When predicting the height of pond for continuous fill, the Somogyi model is very accurate at 200 filling steps/yr, while the UF-McGS model predicts accurately at 30 layers.
11. Sand caps applied as sucharge over waste clays lengthen the time for consolidation.
12. When modelling intermittent fill situations, the UF-

McGS model is more versatile than the Somogyi model, because it allows several filling conditions in one analysis, while the Somogyi model only allows one filling or quiescent condition at a time.

Recommendations

1. The authors recommends expansion of the UF-McGS model into multiple layers, utilizing the model presented herein.
2. Since computer modelling predictions are very susceptible to variation of the material permeability parameters C and D, it is recommended that experimental research be pursued to improve this relationship.

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APPENDIX A
DERIVATION OF VOID RATIOS AND BUOYANT UNIT WEIGHTS USED IN THE UF-McGS PROGRAM

Void Ratio

$$e = \frac{\text{Volume of Voids}}{\text{Volume of Solids}} = \frac{Vv}{Vs}$$

Vv = Volume of Water (Saturated Medium) = Vw

Vs = Volume of Clay (Vc) + Volume of Sand (Vs) + Volume of Other Solids (Vos)

$$\text{Sand/Clay Ratio (SCR)} = \frac{\text{Weight of Sand}}{\text{Weight of Clay}} = \frac{Ws}{Wc}$$

$$\text{Other Solids/Clay Ratio (OSCR)} = \frac{\text{Weight of other Solids}}{\text{Weight of Clay}}$$

$$\text{Solids Content (Sc)} = \frac{Wc}{Wc + Ww}$$

$$e = \frac{Vw}{Vc + Vs + Vos} \tag{B.1}$$

$$\gamma_1 = \frac{Ws}{Vs} = G_1\gamma_w \tag{B.2}$$

$$\gamma_2 = \frac{Wc}{Vc} = G_2\gamma_w \tag{B.3}$$

$$\gamma_3 = \frac{Wos}{Vos} = G_3\gamma_w \tag{B.4}$$

where,

G_1 = specific gravity of sand

G_2 = specific gravity of clay

G_3 = specific gravity of other solids

By rearranging equations B.2, B.3, and B.4 in terms of the volume, and substituting into equation B.1, the following equation may be obtained for the void ratio

$$e = \frac{1 - Sc}{Sc\ \{1/G_2 + SCR/G_1 + OSCR/G_3\}} \tag{B.5}$$

Assuming the total weight of a soil element = 1

$$1 = Ww + Wc + Ws + Wos$$

$$1 = \frac{Wc(1-Sc)}{Sc} + Wc(OSCR) + Wc(SCR) + Wc$$

$$Sc = Wc\ [\ 1 + Sc(OSCR) + Sc(SCR)\ ]$$

Buoyant Stress

The buoyant unit weight may be expressed as

$$\gamma_b = \gamma_t - \gamma_w \tag{B.6}$$

where

γ_t = total unit weight

Also, the total unit weight may be expressed as

$$\gamma_t = \frac{W}{Vw + Vos + Vs + Vc}$$

Rearranging equations B.2, B.3, and B.4 in terms of the volume, and substituting into equation B.6, the following equation may be obtained for the buoyant unit weight

$$\gamma_b = \frac{1}{\dfrac{Sc}{\gamma_w(1+Sc(OSCR)+Sc(SCR))}\left[\dfrac{1-Sc}{Sc} + \dfrac{SCR}{G_1} + \dfrac{OSCR}{G_3} + \dfrac{1}{G_2}\right]} - \gamma_w$$